MULTICRITERIA OPTIMISATION OF RAILWAY TRACK FOR HIGH-SPEED LINES

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ABSTRACT

The paper presents results of multicriteria optimum design of an embedded rail structure (ERS). The optimality criteria are based on the requirements to railway tracks related to cost efficiency of the design, minimum noise emission and minimum wear of wheels and rails. The design variables comprise the material and geometry properties of ERS, namely elastic properties and volume of compound, shape of rails and size of troughs. The static and dynamic behaviour of ERS is analysed using the finite element software ANSYS and RAIL. The multicriteria optimisation problem is solved using preference function approach. The optimisation is performed by a numerical technique that uses Multipoint Approximations based on Response Surface fitting (MARS method). Starting from a conventional embedded rail structure, new designs of ERS suitable for high-speed operations have been obtained. Because of the multicriteria nature of the problem the solution depends on the preferences for the optimal design. The results of the optimisation are presented and discussed in the paper.

INTRODUCTION

Most of the railway tracks used nowadays belong to a traditional ballasted type of track structures. The main drawback of a traditional track is the high costs related to its inspection and maintenance. These costs are considerably increasing for high-speed tracks, which require high positioning accuracy of the rails. One problem of using ballasted tracks for high-speed operations has been reported recently. In particular, due to churning up of ballast particles at high speeds, serious damage of wheels and rails can occur. That is why in recent applications, a railway track design tends more and more towards railway structures without ballast. The major advantage of ballastless tracks is low maintenance effort and high availability.

A ballastless track has lower height and weight compared with a traditional one, which is important for bridges and tunnels. Additionally, recent life cycle studies have shown that from the cost point of view, slab tracks are less expensive and might be very competitive [3]. Also, mechanical properties of a track without ballast can be better determined and therefore the track behaviour can be more accurately described and analysed using numerical methods (e.g. the Finite Element Method).

a b

Figure 1 Shinkansen (a) and Rheda (b) slab track structures

Presently the ballastless track concepts are rapidly developing in a number of countries. The most well known in this respect are the Shinkansen (Japan) and Rheda (Germany) slab track structures (Figure 1) wherein the rails are supported at discrete points. Other type of a track structure without ballast is a so-called Embedded Rail Structure (ERS). In contrast with the Shinkansen and Rheda designs the rails are now continuously supported by elastic compound (Figure 2).

An ERS consists of a continuous reinforced concrete slab rested on a concrete stabilised roadbed, which in turn is placed on a sand base. Two troughs in the slab at 1.5m spacing serve to embed UIC54 rails, a visco-elastic compound, an elastic strip at the bottom of the troughs and some construction utensils.

Within the framework of the STV-project (Stiller TreinVerkeer: Silent Train Traffic), an experimentally developed ERS structure was applied over 150m of the slab at Best. One of the results of the STV project was a new rail profile SA42 (Figure 3) that can reduce the level of acoustic noise by at least 5 dB(A). This result already shows that the efficiency of using conventional rails for ERS is rather arguable and that the performance of ERS can be improved.

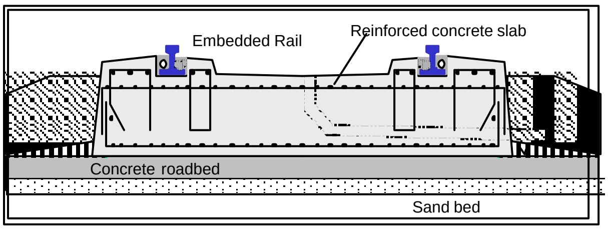

Figure 2 Elements of ERS

In this paper an ERS design is improved using a numerical optimisation technique. First, requirements to an optimum design of ERS are formulated and equivalent response quantities of a structure are obtained using numerical models. The numerical models of ERS used here are based on the Finite Element Method formalism. The static structural responses are evaluated using the ANSYS software and the dynamic behaviour of ERS is analysed using the RAIL program (TU Delft).

An optimum design of an embedded rail structure for high-speed trains has been obtained using a specific optimisation technique, namely Multipoint Approximations based on Response Surface fitting (MARS) [8,12]. The method is iterative and uses mid-range approximation of original objective and constraint functions.

The requirements to an optimum design of ERS and their numerical interpretation are discussed in REQUIREMENTS TO OPTIMUM DESIGN OF ERS. The numerical models of ERS are presented in NUMERICAL MODELS OF ERS. The optimisation procedure is briefly described in OPTIMISATION METHOD. Choice of optimality criteria for ERS and formulation of an optimisation problem are presented in OPTIMISATION OF ERS. Results of optimisation are presented and discussed.

REQUIREMENTS TO OPTIMUM DESIGN OF ERS

The response quantities considered to be important for the optimum performance of ERS used here are summarised below.

- There are many ways to estimate *costs* of ERS. The costs depend to a large extent on the amount of elastic compound used in ERS, which is a quite expensive material. Obviously, to reduce the costs, *volume of elastic compound* should be *minimal*.
- The *'open surface'* of ERS, i.e. the surface of the rail and the elastic compound that is in contact with open air, defines the level of the *acoustic noise*. Obviously, the larger the open surface the more noise is radiating from the structure so that the open surface of ERS should be *minimal* as well.
- An optimum railway track should possess good dynamic properties.
    - Large vibrations of a track can deteriorate passenger's comfort and increase maintenance effort of vehicles, rails and other structures. To avoid the harmful track *vibrations*, resonant frequencies of ERS should not coincide with any of the vehicle resonant frequencies. One way to achieve that is to shift the resonant frequencies of a track as far as possible from the ones of a vehicle. In other words, the first principal *resonant frequency of ERS in vertical direction* should be *maximised*. The resonant frequencies of ERS can be obtained by analysing the Frequency Response Function.
    - Other requirement to optimum track design concerns *acoustic characteristic*, i.e. the ability of a track to damp vibrations before it starts to radiate them via its open surface. A specific response quantity, a so-called *attenuation rate* (or distance damping) characterises the acoustic properties of ERS. The level of the noise radiating from the rails reduces as the attenuation rate of a track structure increases and the frequency range with the high attenuation rates becomes wider [7]. Note that in the latter situation the first *resonant frequency* of the track increases as well.
- Wear of rails defines to large extent *maintenance* costs of a track structure, likewise wear of wheels defines maintenance costs of a vehicle. Here the wheel-rail wear is estimated by analysing wheel-rail contact forces. The standard deviation of the contact forces should be *minimal* in order to reduce the wheel-rail wear [6]. Large variation in vertical track geometry due to the lack of maintenance will inevitably result in high contact forces, which in turn will cause more wheel-rail wear.
- Apart from that, there are three major safety requirements that an optimum ERS design has to satisfy.
    - One requirement concerns the lateral displacements of the rails. Under a specific angular loading condition, the lateral displacements should not exceed a prescribed limit in order to avoid the gauge widening and ultimately train derailment.
    - Other requirement deals with the structural strength, i.e. all components of a track structure have to have enough strength to prevent their damage. Fatigue tests of ERS have shown that from the strength point of view compound is the most vulnerable part of the structure. The strength of compound can be estimated by analysing a distribution of structural stresses. To prevent damage of a track structure the level of stresses should be restricted.
    - One of the advantages of ERS is the lower possibility of lateral buckling. On the other hand there is still a chance that ERS becomes unstable in the vertical direction, especially if there is no adhesion between the rail and

compound. Therefore an optimum design of ERS has to have enough vertical stiffness to prevent its buckling.

All the response quantities of the embedded rail structure mentioned above have been obtained using the Finite Element Method. The static behaviour of ERS has been analysed using ANSYS software whereas for determination of the dynamic responses numerical models implemented in RAIL program have been used. Both static and dynamic numerical models of the embedded rail structure are discussed below.

NUMERICAL MODELS OF ERS

Modelling of static behaviour of ERS

The static response quantities such as stresses and displacements of an embedded rail structure under various loading conditions have been obtained using a general purpose finite element package ANSYS. The 2-D and 3-D FE models of ERS are shown in Figure 3. Before these models have been included in the optimisation process, they were verified by comparing the results of laboratory testing and finite element calculations [9].

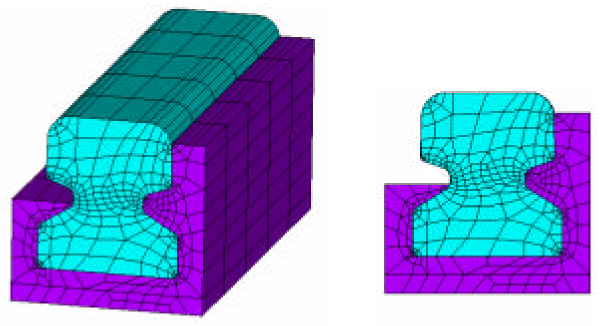

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Figure 3 3-D (a) and 2-D (b) finite element models of ERS with SA42 rail

An elastic strip under the rail, which was a common part in existing designs, is now replaced by elastic compound. In [9] it is demonstrated that that the same behaviour of a structure can be achieved by applying only compound with the adjusted E-modulus and Poisson ratio.

It should be noted that for both 2-D and 3-D finite element models of ERS so-called p-elements have been used. In contrast to traditional h-elements, the use of the p-elements allows to control the calculation error without changing the finite element mesh. Variation of the mesh density can introduce numerical noise in the response function values that slows down the convergence of an optimisation process [8].

Three loading cases have been considered to obtain the static response quantities of a structure for assessment of ERS design.

First, to determine the vertical stiffness of a track the load $F_{y,1} = -30.4\ kN$ has been applied at the top of the railhead as shown in Figure 4a. The static (K_{stat}) and dynamic (K_{dyn}) vertical stiffness are then calculated as

$$K_{stat} = \frac{F_{y,1}}{u_{y,1}} \quad \text{and} \quad K_{dyn} = 2K_{stat}, \qquad (1)$$

where $u_{y,1}$ is the corresponding vertical displacement of the rail.

Since ERS should have enough lateral strength (see the previous section) to prevent train derailment a special loading case has been considered. In this case a concentrated load equivalent to the one of 32.4 kN distributed on 0.5m has been applied to obtain the lateral displacements of the rail $u_{x,2}$ and stresses s in compound. The load has been applied to the rail at 22 degrees relative to the vertical (combined from vertical load $F_{y,2} = -30.4\ kN$ and horizontal load $F_{x,2} = 12.2\ kN$) as shown in Figure 4b. For optimum design the lateral displacements $u_{x,2}$ and maximum (Von Misses) stress $s_{\max}$ should be below their maximum allowable values, i.e.

$$u_{x,2} \le u_{x,allow}, \quad s_{\max} \le s_{allow}. \qquad (2)$$

Here the maximum allowable lateral displacement $u_{x,allow} = 0.002\ m$ has been chosen. The maximum tensile stress at which cracking of compound occurs has been used as s_{allow}. Based on experimental data the following approximation of the maximum tensile stress has been used (the approximation is valid for compound with E-moduli between 1MPa and 10 MPa):

$$s_{tens,\max} = 0.09E + 0.8\,[MPa]. \qquad (3)$$

The maximum allowable stress is then taken as 90% of the maximum tensile stress that reads

$$s_{allow} = 0.9\, s_{tens,\max} \qquad (4)$$

The third loading case concerns upward buckling of a rail. This can happen if there is no (or not enough) adhesion between the rail and compound. The vertical (upward) stiffness is then reduced significantly and depends only on a shape of the rail. For a safe design of ERS the minimum buckling load should not be lower the prescribed value P^*_{bckl}, i.e.

$$P_{bckl} \ge P^*_{bckl} \qquad (5)$$

Here the minimum allowable buckling force $P^*_{bckl} = 1\,MN$ has been used that corresponds to a situation with very high normal forces in rails due to temperature variation and train braking.

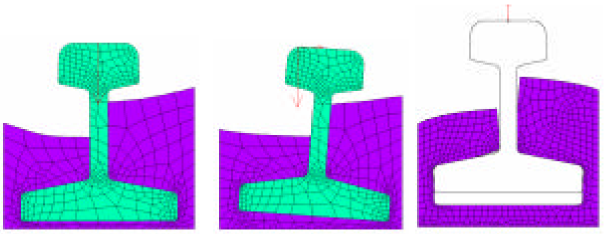
The buckling force P_{bckl} can be estimated analytically by modelling ERS as a beam on an elastic foundation and using the energy approach [11] that reads

$$P_{bckl} = 2\sqrt{EIK_{up}} \tag{6}$$

where EI is the flexure of the rail and K_{up} is the upward stiffness of ERS per unit length.

To estimate the upward stiffness of ERS, the model has been adjusted so that the rail and compound have no common nodes and there is no friction between the rail and compound (Figure 4c). The vertical displacements of the rail are obtained by applying the vertical load $F_{y,3} = 10\ N$ and performing the non-linear contact analysis. The vertical displacements of the rail $u_{y,3}$ are used for the stiffness evaluation as

$$K_{up} = \frac{F_{y,3}}{u_{y,3}\, d}. \tag{7}$$



a b c

Figure 4 Loading cases for static analysis (ANSYS)

Additionally, for each ERS design the following quantities are to be calculated:

- the volume of compound V [dm^3/m] to estimate costs;
- the open surface A [dm^2/m] to estimate acoustic properties of the design.

All the responses of the static model will be used later in this paper to formulate an optimisation problem.

Modelling dynamic behaviour of ERS

RAIL is a finite element program developed at the Railway Engineering Group (TU Delft) for analysis of the dynamic behaviour of railway track structures [5]. Here, the application of RAIL is focussing on two aspects, namely acoustic noise produced by a track and wheel-rail wear.

To estimate the properties of a track related to acoustic noise, the first vertical resonant frequency f_r has been considered. The resonant frequency is obtained by applying an impulse load to ERS and performing the frequency response analysis. The load and all essential input parameters corresponding to this loading case are shown in Figure 5.

Since the level of acoustic noise radiating from ERS depends on both the resonant frequency f_r and the area of the open surface A (see the previous sections) the following formula has been proposed for estimation of the acoustic noise F_N

$$F_N = A\frac{f_{\max} - f_r}{f_{\max} - f_{\min}} \tag{8}$$

where $f_{\min} = 100\ Hz$ and $f_{\max} = 1000\ Hz$ are the lower and upper boundary of the resonant frequency.

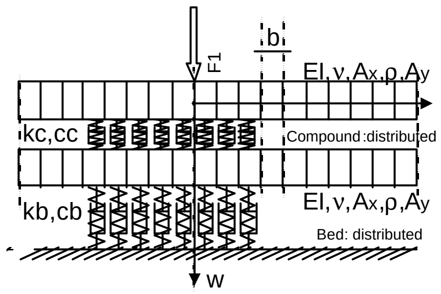


Figure 5 Model of ERS in RAIL (impulse loading case)

A different loading case has been considered to determine the properties of ERS related wheel-rail wear. In the mechanism of wheel-rail wear, variation of wheel-rail contact forces is crucial. The contact forces depend on train and track properties, travelling speed as well as geometry of a track. In this loading case a whole TGV train containing 5 coaches has been modelled as a mass-spring system.

The wheel-rail wear is estimated by the standard deviation of the wheel-rail contact forces W that has been obtained from the 0.8 sec simulation of the TGV train moving with 90 m/s (324 km/h). For details about the model data used for the static and dynamic analyses we refer to [9].

Large values of the E-modulus of compound increase the vertical stiffness of a track. The high stiffness implies the higher resonant frequency of a structure that has positive effect on the reduction of the acoustic noise. On the other hand, the increase of stiffness also implies the increase of the wheel-rail contact forces that has a negative effect on reduction of the maintenance effort due to rail wear. Thus, two conflicting effects occur in the attempt to improve ERS design.

OPTIMISATION METHOD

The method used to optimise ERS is presented below.

General optimisation problem

To make use of numerical optimisation techniques an optimisation problem should be stated in a general form that reads:

Minimise

$$F_0(\mathbf{x}) \rightarrow \min, \quad \mathbf{x} \in R^N \tag{9}$$

subject to

$$F_j(\mathbf{x}) \leq 1, \quad j = 1,\ldots,M \tag{10}$$

and

$$A_i \le x_i \le B_i, \quad i = 1,..., N \qquad (11)$$

where

F_0 is the objective function;

$F_j, \quad j = 1,\dots,M$ are the constraints;

$\boldsymbol{x} = [x_1,... x_N]^T$ is the vector of design variables;

A_i and B_i are the side limits, which define lower and upper bounds of the *i*-th design variable.

Components of the vector $\boldsymbol{x}$ represent various parameters of a structure, such as geometry, material, stiffness and damping properties, which can be varied to improve the design performance. Depending on the problem under consideration the objective and constraint functions (9)-(10) can describe various structural and dynamic response quantities such as weight, reaction forces, stresses, natural frequencies, displacements, velocities, accelerations, etc. Cost, maintenance and safety requirements can be used in the formulation of the optimisation problem as well. The objective function provides a basis for improvement of the design whereas the constraints impose some limitations on behaviour characteristics of a structure.

Approximation concept

The optimisation problem (9)-(11) can be solved using a conventional method of mathematical programming. However, for systems with many degrees of freedom the finite element analysis can be time consuming. As a result, the total computational effort of the optimisation might become prohibitive. This difficulty has been mitigated in the mid-seventies by introducing approximation concepts [1].

According to the approximation concepts the original functions (9)-(10) are replaced with approximate ones which are computationally less time consuming. Instead of the original optimisation problem (9)-(11) a succession of simpler approximated subproblems similar to the original one formulated using the approximation functions is to be solved. Each simplified problem then has the following form:

Minimise

$$\tilde{F}_0^k(\boldsymbol{x}) \to \min, \quad \boldsymbol{x} \in \boldsymbol{R}^{\boldsymbol{N}} \qquad (12)$$

subject to

$$\tilde{F}_j^k(\boldsymbol{x}) \le 1, \quad j = 1,..., M \qquad (13)$$

and

$$A_i^k \le x_i \le B_i^k, \; A_i^k \ge A_i, \; B_i^k \le B_i, \; i = 1,..., N \qquad (14)$$

where the superscript k is the number of the iteration step, $\tilde{F}$ is the approximation of the original function F, A_i^k and B_i^k are *move limits* defining the range of applicability of the approximations.

Since the functions (12)-(13) are chosen to be simple and computationally inexpensive, any conventional method of optimisation [1] can be used to solve the problem (12)-(14). The solution of the problem $\boldsymbol{x}_*^{\boldsymbol{k}}$ is then chosen as starting point for the (k+1)-th step and the optimisation problem (12)-(14) re-formulated with new approximation functions $\tilde{F}_j^{k+1}(\boldsymbol{x}) \le 1, \; (j = 0,..., M)$ and move limits A_i^{k+1} and B_i^{k+1} is to be solved. The process is repeated until the convergence criteria are satisfied.

MARS optimisation technique

The approximation is defined as a function of the design variables $\boldsymbol{x}$ and tuning parameters $\boldsymbol{a}$ (for brevity the indices k and j will be omitted). To determine the components of vector $\boldsymbol{a}$ the following weighted least-squares minimisation problem is to be solved [8,12]:

Find vector $\boldsymbol{a}$ that minimises

$$G(\boldsymbol{a}) = \sum_{p=1}^{P} \{ w_p^{(0)} [F(\boldsymbol{x_p}) - \tilde{F}(\boldsymbol{x}_p, \boldsymbol{a})]^2 \} \qquad (15)$$

Here $F(\boldsymbol{x_p})$ is the value of the original function from (9)-(10)evaluated at the point of the design parameters space $\boldsymbol{x_p}$, and P is the total number of such points; $w_p^{(0)}$ is a weight factor that characterises the relative contribution of the information about the original function at the point $\boldsymbol{x_p}$. For the numerical examples the multiplicative form of the approximating function has been chosen, which has the form

$$\tilde{F}(\boldsymbol{x}) = a_0 \prod_{i=1}^{P} (x_i)^{a_i} \qquad (16)$$

The optimisation process is controlled by changing the move limits in each iteration step. The iteration process is terminated if the approximations are good, none of the move limits is active and the search subregion is small enough. The details about the weight coefficient assignment, the move limits strategy and the most recent developments of the method can be found in [8].

OPTIMISATION OF ERS

As mentioned above, to use a numerical optimisation method a problem should be stated in the form (9) - (11). To optimise ERS, eight geometry parameters and two material parameters of compound have been chosen as the design variables $\boldsymbol{x}$. They are shown in Figure 7.

Note, that the variables x_5 and x_6 represent the filling level of compound relative to the total height of the rail. The lower and upper bounds of the design variables are collected in Table 1.

For an optimum design of ERS the cost, acoustic noise and maintenance (wheel-rail wear) estimated by the amount of compound, by the area of open surface and resonant frequency (8), and by the contact forces respectively should be minimal, i.e.

$$F_C \equiv V \rightarrow \min , \quad F_N \rightarrow \min ,$$
$$F_M \equiv W \rightarrow \min . \tag{17}$$

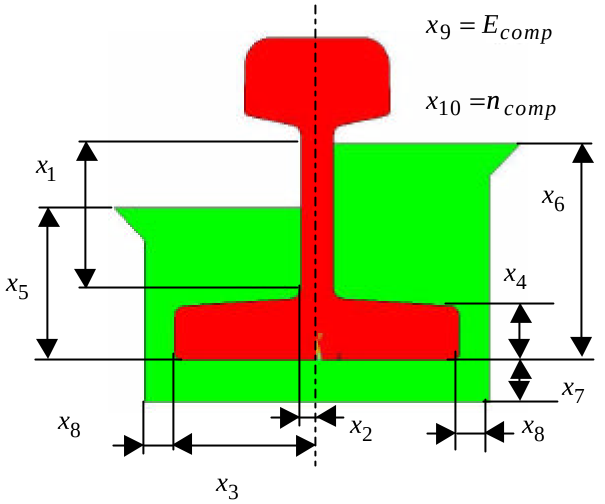


Figure 7 Design variables of the optimisation problem

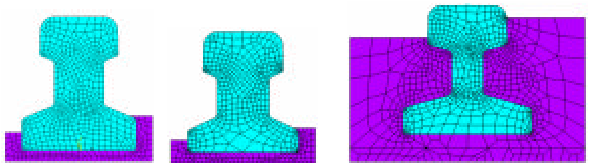
Design variable	**Lower bound**	**Upper bound**	**Initial value**	**Units**
x_1	8.9	85.0	80.0	mm
x_2	8.0	36.0	8.0	mm
x_3	36.0	70.0	70.0	mm
x_4	15.0	30.0	25.0	mm
x_5	0.100	0.750	0.471	-
x_6	0.100	0.900	0.669	-
x_7	5.0	15.0	10.0	mm
x_8	15.0	60.0	15.0	mm
x_9	1.0	10.0	5.6	MPa
x_{10}	0.40	0.46	0.46	-

Table 1 Side limits and initial values of design parameters

The requirements preventing damage of ERS, train derailment (2) and buckling of rails (6) have been used as the constraints:

$$F_1(\boldsymbol{x}) \equiv \frac{u_{x,2}}{u_{x,allow}} \leq 1, \quad F_2(\boldsymbol{x}) \equiv \frac{s}{s_{\ allow}} \leq 1 ,$$
$$F_3(\boldsymbol{x}) \equiv \frac{P^*_{bckl}}{P_{bckl}} \leq 1 . \tag{18}$$

Thus, the optimisation problem (17)-(18) has three objectives. Such problems are characterised by an objective conflict, i.e. when improving the values of one objective at least one other objective deteriorates. A typical approach to solve a multiobjective problem is to convert it to a single objective one [1]. One way is to consider the most important function as the only objective and to impose limits on the other objective functions (to treat them as the constraints) [2].



a b c

Figure 6 Results of single criterion optimisations (a - compound, b - noise, c - maintenance)

The other way is to make a composition of the objective functions while assigning weights (preferences) for each function that reflect the relative importance of the objectives. The solution of the composition problem is a compromise solution since it depends on the weight factors. The whole set of the compromised solutions is called the Pareto set. In practice, a representative subset of the Pareto set is to be determined so that a person in charge (a decision maker) can choose a best appropriate solution [10]. The latter approach has been used here to solve the optimisation problem (17)-(18). Thus, the following objective function has been constructed

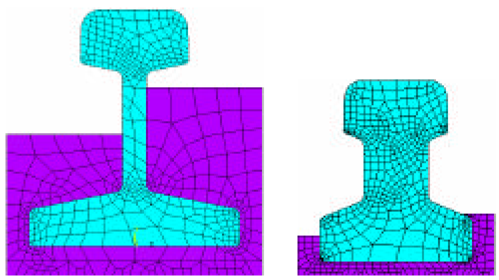
$$F_0 \equiv w_C \frac{F_C}{F_{C,ut}} + w_N \frac{F_N}{F_{N,ut}} + w_M \frac{F_M}{F_{M,ut}} \rightarrow \min \tag{19}$$

where

w_C, w_N and w_M are the weight coefficients ($w_C + w_N + w_M = 1$) reflecting the relative importance of compound, noise and maintenance reduction in a final design of ERS;

$F_{C,ut}, F_{N,ut}$ and $F_{M,ut}$ are the normalising coefficients since the objective functions have different dimensions. The normalising coefficients are obtained as the solutions (objectives) of the corresponding single criterion optimisation problems from (17), the so-called *utopian* solutions since they could never be obtained in the multicriteria optimisation.

The initial design is shown in Figure 8a and the results of the single criterion optimisations are given in Figure 6. The numerical results are collected in Table 2. From these results it can be seen that the optimum design of the single optimisation problem with respect to the cost



a b

Figure 8 Initial design (a) and results of multicriteria optimisation with equal weights (b) and noise (Figure 6a and Figure 6b) are more close to each other as to compare with the result of the maintenance optimisation (Figure 6c). This can be explained by the fact that the compound and noise objectives are not conflicting very much. Both designs have quite good acoustic properties (improved response frequency 350 Hz and 380 Hz respectively and almost a double reduction of the open area A) while they are poor with respect to maintenance properties (high contact forces 14.1 kN and 15.1 kN).

During the third single criterion optimisation the properties related to maintenance of ERS have been improved dramatically (contact forces of 17.5 kN for the initial design and 10.0 for the optimised one). However, the acoustic properties are much worse (noise objective 1.09) than the ones of the optimal designs of the two first single optimisations (noise objective 0.54 and 0.46 respectively). Moreover, the amount of compound in the optimum design of the maintenance optimisation has become even larger (19.973 dm^3/m) than the one in the initial design (13.536 dm^3/m).

The small amount of compound above the rail (Figure 6a and Figure 6b) was enough to prevent upward buckling since the stiffness of the rail, which is important for determination of the buckling force according to the chosen analytical criteria (5)-(6), has increased. More accurate calculation of the buckling force using the non-linear numerical analysis might be useful in future optimisation.

A number of multicriteria optimisation problems with different sets of the preference coefficients have been solved. The preference factors can reflect different points of view on optimum design of ERS (point of view of society, investor, maintenance contractor etc). Based on results of the optimisations a decision about a final design can be made.

In the first optimisation it is assumed that all the objectives are equally important. The optimum design is shown in Figure 8b. The numerical results are collected in Table 2. Figure 9 shows the results of optimisation problems with varied preference factors related to the reduction of the maintenance effort and the noise (see Table 2) while considering the cost objective less important (the corresponding preference coefficient is constant $w_C = 0.1$). The numerical results clearly reflect the effect of the chosen preferences for the optimum design. From Table 2 it can be seen that the stiffness of compound and resonant frequency of ERS are increasing (from 5.39 MPa to 9.25 and from 290 Hz to 380 Hz respectively) as the noise objective becomes

	Initial design	Single criterion optimisation			Multicriteria optimisation				
		Comp.	**Noise**	**Maint.**	**Wc=0.33, Wn=0.33, Wm=0.33**	**Wc=0.1, Wn=0.1, Wm=0.8**	**Wc=0.1, Wn=0.4, Wm=0.5**	**Wc=0.1, Wn=0.6, Wm=0.3**	**Wc=0.1, Wn=0.8, Wm=0.1**
Design variables									
x_1[mm]	80.0	40.9	35.4	39.4	41.2	36.4	33.8	35.7	35.7
x_2[mm]	8.0	22.4	23.2	13.7	22.3	27.2	25.0	19.6	19.6
x_3[mm]	70.0	52.5	50.1	58.6	52.7	51.0	49.4	50.2	50.2
x_4[mm]	25.0	29.9	19.5	23.6	29.7	17.7	15.9	20.1	20.1
x_5	0.471	0.136	0.100	0.748	0.140	0.102	0.100	0.100	0.100
x_6	0.669	0.257	0.200	0.900	0.261	0.201	0.183	0.203	0.203
x_7[mm]	10.0	5.0	5.0	12.8	5.0	5.0	5.3	5.3	5.0
x_8[mm]	15.0	15.0	15.0	48.1	15.0	16.4	15.0	15.0	15.0
x_9[MPa]	5.60	9.44	9.99	4.13	6.86	5.39	7.16	9.25	9.25
x_{10}	0.46	0.43	0.49	0.41	0.46	0.44	0.44	0.43	0.43
Constraints									
s [MPa]	1.07	1.15	2.24	0.91	1.14	1.09	1.14	1.18	1.14
$u_{x,2}$ [mm]	1.83	1.08	1.38	1.61	1.44	1.72	1.30	1.06	1.03
P_{bckl} [MN]	12.5	1.7	1.1	5.4	1.7	1.2	1.2	1.4	1.4
Objectives									
F_C [dm^3/m]	13.536	2.103	1.806	19.973	2.126	1.911	1.816	1.886	1.820
A [dm^2/m]	134.8	74.3	67.5	120.7	74.9	67.5	65.3	69.3	69.2
f_r [Hz]	250	350	380	190	310	290	320	350	380
F_N	1.12	0.54	0.46	1.09	0.57	0.53	0.49	0.50	0.48
F_M [kN]	17.5	14.1	15.1	10.0	12.3	11.2	12.4	13.5	13.8

Table 2 Numerical results of optimisations

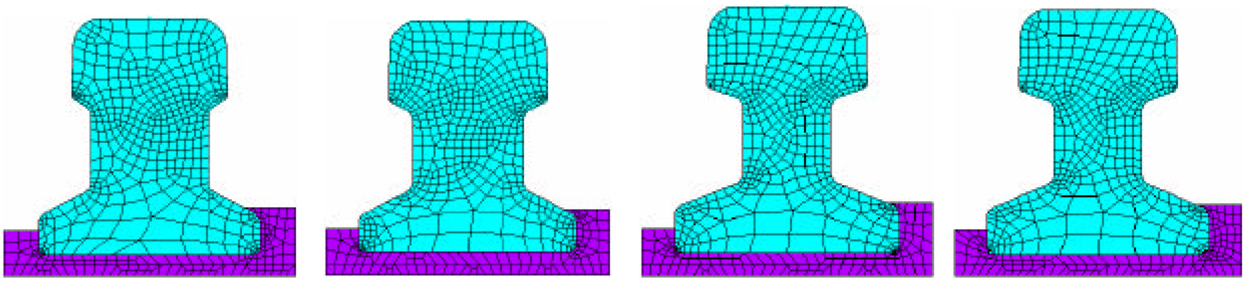

a b c d

Figure 9 Results of multicriteria optimisation 2 (a), 3 (b), 4 (c) and 5 (d) from Table 2

more important (w_N increases from 0.1 to 0.8). On the other hand the contact forces are increasing as well (from 11.2 kN to 13.8 kN) as the maintenance becomes less important (w_M decreases from 0.8 to 0.1). From Figure 9 one can see the tendency in the changing of the rail shape of the optimum design as the maintenance objective becomes less important (from a to d). It also should be noted that the only difference between the two last designs (c and d) is the thickness of the compound layer under the rail. Other compromised solutions can be found by assigning different preference factors for the objectives and performing a similar optimisation.

CONCLUSIONS

A procedure for design of railway track structures using numerical techniques has been presented. The procedure has been applied to optimal design of an Embedded Rail Structure. Both static and dynamic behaviour of ERS have been analysed using finite element methods, while the optimum search has been directed by an advanced optimisation technique.

The multiobjective optimisation problem has been solved by transforming it to a single objective one. The single objective is composed from the original using weight factors (preferences) that reflect relative importance of the objectives. To prevent numerical difficulties the value of the objective functions have been normalised using the results of the single optimisations (utopian solutions) that have been performed beforehand.

Several optimal designs of ERS have been obtained using different sets of the preference coefficients. Other solutions can be obtained by performing the optimisation with different preferences for the optimum design, which help to make a decision about the final design.

REFERENCES

1. Barthelemy, J.-F.M., Haftka, R.T. (1993) Approximation Concept for Optimum Structural Design – a Review. *Structural Optimization*, **5**, 129-144.
2. Eschenauer, H.A. (1999) Multidisciplinary engineering strategies for product and process development - modelling, simulation, optimization, applications. In: V.V. Toropov (Ed.) Engineering Design Optimization, Product and Process Improvement. *Proc. of the Ist ASMO UK/ISSMO conference on Engineering Design Optimization*, Ilkley, West Yorkshire, UK, July 8-9, 1999.
3. Esveld, C. (1999) Slab Track: A competitive solution. Rail International, Schienen der Welt, June 1999.
4. Esveld, C., Kok, A.W.M. (1998) Interaction between Moving Vehicles and Railway Track at High Speed. Rail Engineering International, **27**, 3, 14-16.
5. Kok, A.W.M. (1998) Moving loads and vehicles on a rail track structure: RAIL User's Manual, Report 0321-12202, TU Delft.
6. Man, A.P. de (1999) Interim Report Dynatrack: 2 years of research on the dynamic behaviour of railway track structures (in Dutch), *Report 7-99-111-10*, Railway Engineering Group, TU Delft. ISSN 0169-9288.
7. Markine, V.L., Man, A.P. de, Esveld, C. (1998) A Procedure for Design and Optimization of a Railway Track Structure. *Proc. Interactive Conference: Cost Effectiveness and Safety Aspects of Railway Track.* Union Internationale des Chemins de Fer - 8-9 December, Paris.
8. Markine, V.L. (1999) *Optimization of the Dynamic Behaviour of Mechanical Systems.* PhD Thesis, TU Delft. Shaker Publishing B.V. ISBN 90-423-0069-8.
9. Markine, V.L., de Man, A.P., Jovanovic, S., Esveld, C. (1999) Modelling and Optimisation of an Embedded Rail Structure. *Proceedings of Innovations in the design and assessment of railway track.* International conference, Trade Show and Workshops. December 2-3, 1999, Delft University of Technology, Delft, The Netherlands.
10. Stadler, W. (1988) Multicriteria Optimization in Engineering and in the Science. Plenum Press, New York, London.
11. Timoshenko S.P. (1936) *Theory of Elastic Stability*. McGraw-Hill, New York.
12. Toropov, V.V., Filatov, A.A., Polynkin, A.A. (1993) Multiparameter Structural Optimization using FEM and Multipoint Explicit Approximations. *Structural Optimization*, **6**, 7-14.