

OPTIMUM DESIGN OF EMBEDDED RAIL STRUCTURE FOR HIGH-SPEED LINES

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ABSTRACT

The paper presents some key points of multi criteria optimum design of an embedded rail structure (ERS) such as numerical modeling of track behaviour, choice of optimality criteria, formulation of an optimization problem. The optimality criteria are based on the requirements to railway tracks related to cost efficiency of the design, minimum noise emission and minimum wear of rails and wheels. The design variables comprise the material and geometry properties of ERS, namely elastic properties and volume of compound, shape of rails and size of troughs. The static and dynamic behaviour of Embedded Rail Structure is analyzed using the finite element software ANSYS and RAIL. The multi criteria optimization problem is solved using preference coefficient approach. The optimization is performed by a numerical technique that uses Multipoint Approximations based on Response Surface fitting (MARS method). Starting from a conventional embedded rail structure, two designs of ERS suitable for high-speed operations have been obtained. Because of the multi criteria nature of the problem the solutions depend on the preferences for the optimal design. The results of the optimization are presented and discussed in the paper.

Introduction

Most of the railway tracks used nowadays belong to a traditional ballasted type of track structures. The main drawback of a traditional track is the high costs related to its inspection and maintenance. These costs are considerably increasing for high-speed tracks, which require high positioning accuracy of the rails. One problem of using ballasted tracks for high-speed operations has been reported recently. In particular, due to churning up of ballast particles at high speeds, serious damage of wheels and rails can occur. That is why in recent applications, a railway track design tends more and more towards railway structures without ballast [4]. The major advantage of ballastless tracks is low maintenance effort and high availability. Ten year of experience of Deutsche Bahn AG (Germany) with high-speed operation shows that slab track installed on these lines requires practically no maintenance and exhibits no position changes [12].

A ballastless track has lower height and weight compared with a traditional one, which is important for bridges and tunnels. Additionally, recent life cycle studies have shown that from the cost point of view, slab tracks are less expensive and might be very competitive [5]. Also, mechanical properties of a track without ballast can be better determined and therefore the track behaviour can be more accurately described and analyzed using numerical methods (e.g. the Finite Element Method). As a result, modern numerical optimization techniques can be applied to obtain an optimum design of a railway track.



Figure 1 Shinkansen (a) and Rheda (b) slab track structures

Presently the ballastless track concepts are rapidly developing in a number of countries. The most well known in this respect are the Shinkansen (Japan) and Rheda (Germany) slab track structures (Figure 1) wherein the rails are supported at discrete points. Another type of a track structure without ballast is a so-called Embedded Rail Structure (ERS). In the mid-sixties, Nederlandse Spoorwegen (NS) in the Netherlands has started to use such a structure on bridges. Since 1976, NS is using ERS for level crossings. By that time also ERS on a concrete slab has been introduced. In contrast with the Shinkansen and Rheda designs the rail is now continuously supported by elastic compound embedding the rails.

Recently a track of 3km length with ERS on a concrete slab (Figure 2) has been built on the south of the Netherlands near Best. This structure consists of a continuous reinforced concrete slab rested on a concrete stabilized roadbed, which in turn is placed on a sand base. Two troughs in the slab at 1.5m spacing serve to embed UIC54 rails, a visco-elastic compound (Corkelast®¹), an elastic strip at the bottom of the troughs and some construction utensils. The great advantage of such a structure is that the rail positioning is carried out "top-down" which means that the final track geometry is hardly influenced by the geometry of the supporting slab.

Within the framework of the STV-project (Stiller TreinVerkeer: Silent Train Traffic), an experimentally developed ERS structure was applied over 150m of the slab at Best. One of the results of the STV project was a new rail profile SA42 (Figure 3) that can reduce the level of acoustic noise by at least 5 dB(A). This result already showed that the efficiency of using conventional rails for ERS was rather arguable and that the performance of ERS could be improved.

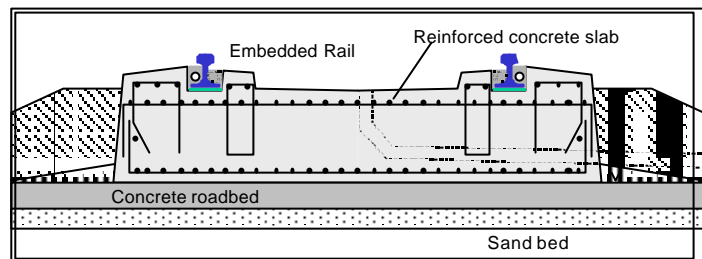


Figure 2 ERS near Best (the Netherlands)

In the traditional design of engineering structures the optimization is performed in a primitive way by modifying design parameters and repeated numerical analyses. The modifications are mostly based on the designer's experience and possibly on the structural sensitivity information. However, it is a time consuming process, especially when the number of the design variables is large. Moreover, the success can not be guaranteed.

The most systematic way to improve a design is to use a numerical optimization technique. In this paper some key points of numerical optimization of ERS are discussed. They concern:

- numerical modeling of various behaviour of a track;
- choice of design variables;
- choice of optimality criteria;
- formulation of an optimization problem.

First, requirements to an optimum design of ERS are to be formulated. To use a numerical optimization technique they should be transformed to equivalent response quantities of a structure, which can be obtained using numerical models. The numerical models of ERS used here are based on the Finite Element Method formalism. The static structural responses are evaluated using the ANSYS software and the dynamic behaviour of ERS is analyzed using the RAIL program (TU Delft). To be used in the optimization relations between the responses of the 2-D and 3-D static models have been investigated beforehand and the static and dynamic models have been coupled to each other.

An optimum design of an embedded rail structure for high-speed trains has been obtained using a specific type of optimization technique, namely Multipoint Approximations based on Response Surface fitting (MARS) [10,15]. The method is iterative and uses mid-range approximation of original objective and constraint functions. The method is implemented in the software IMOPT. For optimization, the ANSYS and RAIL software have been externally coupled with the optimization package.

¹ Corkelast is a registered trademark of Edilon b.v.

The requirements to an optimum design of ERS and their numerical interpretation are discussed in [REQUIREMENTS TO OPTIMUM DESIGN OF ERS](#). The numerical models of ERS are presented in [NUMERICAL MODELS OF ERS](#). The optimization procedure is briefly described in [OPTIMIZATION METHOD](#). Choice of optimality criteria for ERS and formulation of an optimization problem are presented in [OPTIMIZATION OF ERS](#). Results of optimization are presented and discussed.

REQUIREMENTS TO OPTIMUM DESIGN OF ERS

The optimization process commences from definition of an optimum design. In this stage a decision about requirements to the best performing railway track is to be made. These requirements are then to be transformed to equivalent response quantities of a track structure. The response quantities – evaluated numerically – are used in the optimization process. Here the following response quantities are considered to be important for the optimum performance of ERS:

- A. There are many ways to estimate *costs* of ERS. The costs depend to a large extent on the amount of the elastic compound used in ERS, which is a quite expensive material. Obviously, to reduce the costs, *volume of elastic compound* should be *minimal*.
- B. The ‘*open surface*’ of ERS, i.e. the surface of the rail and the elastic compound that is in contact with open air, defines the level of the *acoustic noise*. Obviously, the larger the open surface the more noise is radiating from the structure so that the open surface of ERS should be *minimal* as well.
- C. An optimum railway track should possess good dynamic properties. Large vibrations of a track can deteriorate passenger’s comfort and increase maintenance effort of vehicles, rails and other structures. To avoid the harmful track *vibrations*, resonant frequencies of ERS should not coincide with any of the vehicle resonant frequencies. Amplification of the structural response in the neighbourhood of these frequencies should be prohibited [2]. One way to achieve that is to shift the resonant frequencies of a track as far as possible from the ones of a vehicle. In other words, the first principal *resonant frequency of ERS in vertical direction* should be *maximized*. The resonant frequencies of ERS can be obtained by analyzing the Frequency Response Function.
- D. Other requirement to optimum track design concerns *acoustic characteristic*, i.e. the ability of a track to damp vibrations before it starts to radiate them via its open surface. A specific response quantity, a so-called *attenuation rate* (or distance damping) characterizes the acoustic properties of ERS. The level of the noise radiating from the rails reduces as the attenuation rate of a track structure increases and the frequency range with the high attenuation rates becomes wider [9]. Note that in the latter situation the first *resonant frequency* of the track increases as well.
- E. Wear of rails defines to large extent *maintenance costs* of a track structure, likewise wear of wheels defines maintenance costs of a vehicle. Here the wheel-rail wear is estimated by analyzing wheel-rail contact forces due to rail surface irregularities. The standard deviation of the *wheel-rail contact forces* should be *minimal* in order to reduce the wheel-rail wear [8]. Large variation in vertical track geometry due to the lack of maintenance will inevitably result in high contact forces, which in turn will cause more wheel-rail wear.
- F. Apart from that, there are three major safety requirements that an optimum ERS design has to satisfy. One requirement concerns the lateral displacements of the rails. Under a specific angular loading condition, the lateral displacements should not exceed a prescribed limit in order to avoid the gauge widening and ultimately train derailment.
- G. Other requirement deals with the structural strength, i.e. all components of a track structure have to have enough strength to prevent their damage. Fatigue tests of ERS have shown that from the strength point of view compound is the most vulnerable part of the structure. The strength of compound can be estimated by analyzing a distribution of structural stresses. To prevent damage of a track structure the level of stresses should be restricted.
- H. One of the advantages of ERS is absence of possibility of lateral buckling. On the other hand there is still a chance that ERS becomes unstable in the vertical direction, especially if there is no adhesion between the rail and compound. Therefore an optimum design of ERS has to have enough vertical stiffness to prevent its buckling.

All the response quantities of the embedded rail structure mentioned above have been obtained using the Finite Element Method. The static structural behaviour has been analyzed using ANSYS software whereas for determination of the dynamic responses numerical models implemented in RAIL program

have been used. Both static and dynamic numerical models of the embedded rail structure are discussed below.

NUMERICAL MODELS OF ERS

Modeling of ERS using ANSYS software

The static response quantities such as stresses and displacements of an embedded rail structure under various loading conditions have been obtained using a general purpose finite element package ANSYS. The 2-D and 3-D FE models of ERS are shown in Figure 3. Before these models have been included in the optimization process, they were verified by comparing the results of laboratory testing and finite element calculations [11].

An elastic strip under the rail, which was a common part in existing designs, is now replaced by elastic compound. In [11] it is demonstrated that that the same behaviour of a structure can be achieved by applying only compound with adapted E-modulus and Poisson ratio.

Although the 3-D model (Figure 3a) can accurately describe behaviour of ERS, due to high computational cost, it was practically impossible to use it in an optimization process where multiple evaluations of structural responses are required. That is why the 2-D model has been created (Figure 3b) assuming that in the middle of an ERS sample the model satisfies a plane strain condition. For the 2-D model, 8-nodes solid elements (thickness $d = 0.01 \text{ m}$) have been used. The results of the 2-D model have been compared with those obtained using the 3-D model [11].

It should be noted that for both 2-D and 3-D finite element models of ERS so-called p-elements have been used. In contrast to traditional h-elements, the use of the p-elements allows to control the calculation error without changing the finite element mesh. Variation of the mesh density can introduce numerical noise in the response function values that slows down the convergence of an optimization process [10].

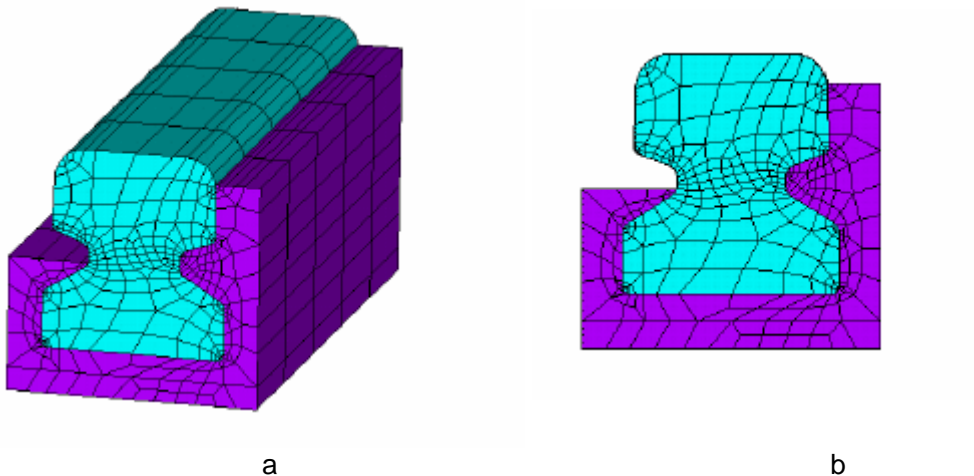


Figure 3 3-D (a) and 2-D (b) finite element models of ERS with SA42 rail

Input and output of static model

In order to analyze the static behaviour of embedded rail structure in ANSYS the numerical model requires the input of the following parameters:

- Geometry of ERS, i.e. the exact shape of the rail, trough and level of compound.
- Material properties of the rail and compound. For the rail well-known steel material properties have been used. For compound, both E-modulus and Poisson ratio are parameters to be optimized.
- Applied loads depending on a loading case.

Three loading cases have been considered to obtain the static response quantities of a structure for assessment of ERS design. First, to determine the vertical stiffness of a track the load $F_{y,1} = -30.4 \text{ kN}$ has been applied at the top of the railhead as shown in Figure 4a. The static (K_{stat}) and dynamic (K_{dyn}) vertical stiffnesses are then calculated as

$$K_{stat} = \frac{F_{y,1}}{u_{y,1}} \quad \text{and} \quad K_{dyn} = 2 K_{stat}, \quad (1)$$

where $u_{y,1}$ is the corresponding vertical displacement of the rail.

Since ERS should have enough lateral strength (see the previous section) to prevent train derailment a special loading case has been considered. In this case a concentrated load equivalent to the one of 32.4 kN distributed on 0.5m has been applied to obtain the lateral displacements of the rail $u_{x,2}$ and stresses \mathbf{s} in compound. The load has been applied to the rail at 22 degrees relative to the vertical (combined from vertical load $F_{y,2} = -30.4 \text{ kN}$ and horizontal load $F_{x,2} = 12.2 \text{ kN}$) as shown in Figure 4b. For optimum design the lateral displacements $u_{x,2}$ and maximum (Von Misses) stress \mathbf{s}_{max} should be below their maximum allowable values, i.e.

$$u_{x,2} \leq u_{x,allow}, \quad \mathbf{s}_{max} \leq \mathbf{s}_{allow}. \quad (2)$$

Here the maximum allowable lateral displacement $u_{x,allow} = 0.002 \text{ m}$ has been chosen. The maximum tensile stress at which cracking of compound occurs has been used as \mathbf{s}_{allow} . Based on experimental data the following approximation of the maximum tensile stress has been used (the approximation is valid for compound with E-moduli between 1MPa and 10 MPa):

$$\mathbf{s}_{tens,max} = 0.09E + 0.8 \text{ [MPa]}. \quad (3)$$

The maximum allowable stress is then taken as 90% of the maximum tensile stress that reads

$$\mathbf{s}_{allow} = 0.9 \mathbf{s}_{tens,max} \quad (4)$$

The third loading case concerns upward buckling of a rail. This can happen if there is no (or not enough) adhesion between the rail and compound. The vertical (upward) stiffness is then reduced significantly and depends only on a shape of the rail. For a safe design of ERS the minimum buckling load should not be lower the prescribed value P_{bckl}^* , i.e.

$$P_{bckl} \geq P_{bckl}^* \quad (5)$$

Here the minimum allowable buckling force $P_{bckl}^* = 1 \text{ MN}$ has been used that corresponds to a situation with very high normal forces in rails due to temperature variation and train braking.

The buckling force P_{bckl} can be estimated analytically by modeling ERS as a beam on an elastic foundation and using the energy approach [14] that reads

$$P_{bckl} = 2\sqrt{EI K_{up}} \quad (6)$$

where EI is the flexure of the rail and K_{up} is the upward stiffness of ERS per unit length.

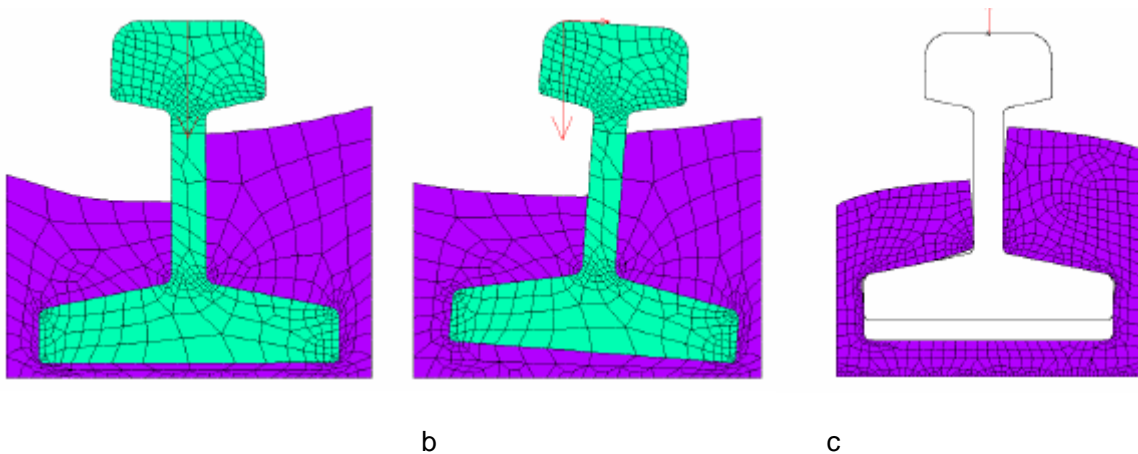


Figure 4 Loading cases for static analysis (ANSYS)

To estimate the upward stiffness of ERS, the model has been adjusted so that the rail and compound have no common nodes and there is no friction between the rail and compound (Figure 4c). The vertical displacements of the rail are obtained by applying the vertical load $F_{y,3} = 10 \text{ N}$ and performing the non-linear contact analysis. The vertical displacements of the rail $u_{y,3}$ are used for the stiffness evaluation as

$$K_{up} = \frac{F_{y,3}}{u_{y,3} d} \quad (7)$$

Additionally, for each ERS design the following quantities are to be calculated:

- the volume of compound V [dm^3/m] to estimate costs;
- the open surface A [dm^2/m] to estimate acoustic properties of the design.

All the responses of the static model will be used later in this paper to formulate an optimization problem.

Modeling the dynamic behaviour of ERS in RAIL

RAIL is a finite element program developed at the Railway Engineering Group (TU Delft) for analysis of the dynamic behaviour of railway track structures [7]. Here, the application of RAIL is focusing on two aspects, namely acoustic noise produced by a track and wheel-rail wear.

Model and method

For analysis of ERS in finite element programs a correct description of the model is essential. RAIL can handle structures in 2D-plane with properties in vertical direction, which means that:

- track has to be simplified to one beam (generally one rail) which is discretely or continuously supported;
- loads applied to a track are only in vertical direction;
- geometry of a track is only to be defined in vertical plane;
- vehicle properties are restricted to the major components such as wheels, bogies and bodies connected to each other by springs and dampers in order to describe a train.

To estimate the properties of a track related to acoustic noise, the first vertical resonant frequency f_r should be considered. The resonant frequency is obtained by applying an impulse load to ERS and performing the frequency response analysis. The load and all essential input parameters corresponding to this loading case are shown in Figure 5.

A different loading case should be considered to determine the properties of ERS related wheel-rail wear. In the mechanism of wheel-rail wear, variation in wheel-rail contact forces is crucial. The contact forces depend on train and track properties, traveling speed as well as geometry of a track.

Input and output of the dynamic model

Input of the dynamic model for both loading cases consists of:

- rail: geometry is subject to change during the optimization process. The rail head is similar to UIC54 and its shape does not change;
- trough: geometry is subject to change during the optimization process. There should be at least 15mm space between the rail and trough on both sides of the rail foot;
- compound: dynamic stiffness is according to (1). Damping is calculated assuming the critical damping ratio of 8 %;
- slab dimensions: 0.9m x 0.35m with a single trough on top of it (for one rail);
- slab material: reinforced concrete (quality B35);
- subgrade: stiffness 100 MN/m/m', damping 15 kNs/m/m';

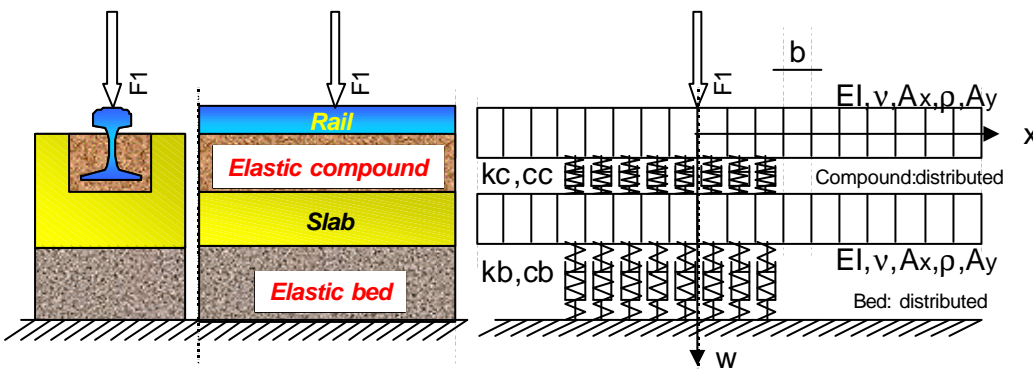


Figure 5 Model of ERS in RAIL (impulse loading case)

The resonant frequency f_r is calculated using the following data:

- impulse load (delta function within 0.2 milliseconds) with the amplitude 1 kN is applied in the middle of the model;
- rail (beam) element length is 0.05m, total model length is 40m;
- simulation time and accuracy: 0.5 seconds with time steps of 0.1 milliseconds;
- recording positions: driving point (force application point) and point at the distances 0.25, 0.5, 1, 2, 4, 8 and 16m relative to the driving point;
- resonant frequency is reported at the maximum of the frequency response function (FRF) in Hz.

Since the level of acoustic noise radiating from ERS depends on both the resonant frequency f_r and the area of the open surface A (see the previous sections) the following formula has been proposed for estimation of the acoustic noise F_N

$$F_N = A \frac{f_{\max} - f_r}{f_{\max} - f_{\min}} \quad (8)$$

where $f_{\min} = 100 \text{ Hz}$ and $f_{\max} = 1000 \text{ Hz}$ are the lower and upper boundary of the resonant frequency.

To obtain wheel-rail contact forces (standard deviation) W the following data have been used:

- TGV train containing 5 coaches (2 locomotives and 3 passenger-cars), a wheel load is 85 kN;
- train speed of 90 m/s (324 km/h);
- rail (beam) element is 0.05m, total model length is 250m;
- simulation time and accuracy: 0.8 seconds with time steps of 4 milliseconds;
- recording wheel-rail contact forces of the front wheel;
- contact forces are stored for the full 0.8 seconds running time for each time point. Based on the results of the last 0.7 seconds (63m of train running) the standard deviation of the contact forces W is determined.

Vertical rail geometry is irregular. Here, it is modeled using summation of periodic (sinus) functions with wavelengths ranging between 0.1m and 10m and random phase shifts. At 90 m/s these wavelengths cause the vibrations between 9Hz and 900Hz, which sufficiently cover the high frequent contact forces between wheel and rail. Amplitude of each periodic function used for approximation of the vertical track geometry is given by the expression:

$$A_i = s \sqrt{\frac{I_i}{I_0}}, \quad (9)$$

where

I_i [m] is the wavelength of the i -th periodic function;

A_i [mm] is the amplitude of the i -th periodic function;

$I_0 = 1.0 \text{ m}$ is the reference wavelength;

$s = 0.01 \text{ mm}$ is the scaling factor used for all wavelengths to achieve a required standard deviation of the total rail geometry. The standard deviation of the rail geometry for the waveband 0.1-10m is 0.25mm. This value corresponds to a good quality track for high-speed operations.

Large values of the E-modulus of compound increase the vertical stiffness of a track. The high stiffness implies the higher resonant frequency of a structure that has positive effect on the reduction of the acoustic noise. On the other hand, the increase of stiffness also implies the increase of the wheel-rail contact forces that has a negative effect on reduction of the maintenance effort due to rail wear. Thus, two conflicting effects occur in the attempt to improve ERS design.

OPTIMIZATION METHOD

The requirements to an optimum design of ERS and the corresponding structural responses discussed in the previous sections will be used later in the paper to formulate an optimization problem. The basics of the optimization theory and the MARS optimization technique used here are briefly presented below.

General optimization problem

To make use of numerical optimization techniques an optimization problem should be stated in a general form that reads:

Minimize

$$F_0(x) \rightarrow \min, x \in R^N \quad (10)$$

subject to

$$F_j(x) \leq 1, j = 1, \dots, M \quad (11)$$

and

$$A_i \leq x_i \leq B_i, i = 1, \dots, N \quad (12)$$

where

F_0 is the *objective function*;

$F_j, j = 1, \dots, M$ are the *constraints*;

$x = [x_1, \dots, x_N]^T$ is the vector of *design variables*;

A_i and B_i are the so-called *side limits*, which define lower and upper bounds of the i -th design variable.

Components of the vector x represent various parameters of a structure, such as geometry, material, stiffness and damping properties, which can be varied to improve the design performance. Depending on the problem under consideration the objective and constraint functions (10)-(11) can describe various structural and dynamic response quantities such as weight, reaction forces, stresses, natural frequencies, displacements, velocities, accelerations, etc. Cost, maintenance and safety requirements can be used in the formulation of the optimization problem as well. The objective function provides a basis for improvement of the design whereas the constraints impose some limitations on behaviour characteristics of a structure.

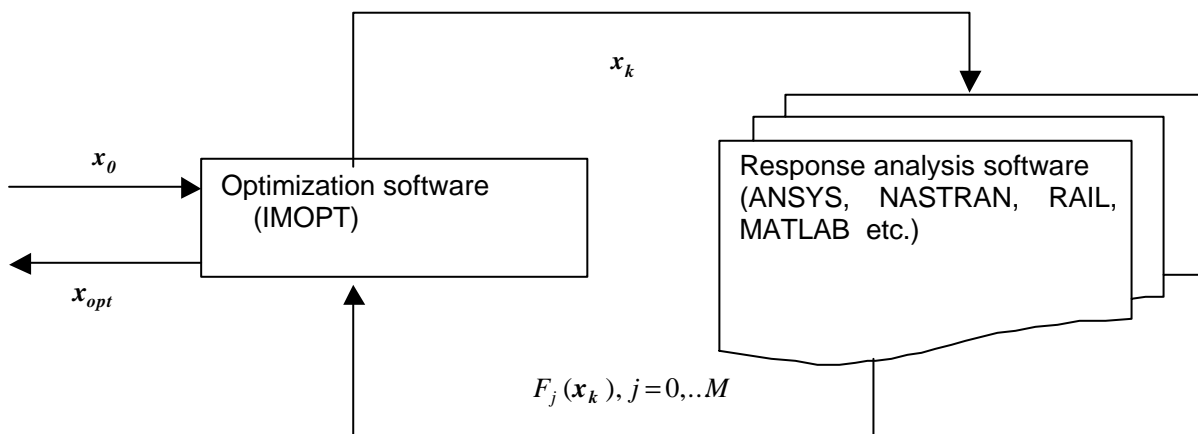


Figure 6 Computer implementation of optimization method

Solution of an optimization problem is an iterative process, which involves multiple evaluations of the objective and constraint functions (10)-(11) in the points of the design variable space assigned by an optimization method. Typically, the values of the functions can be obtained using one of the numerical methods, e.g. the Finite Element Method.

Computer implementation of the optimization method used here is shown in Figure 6. Starting from the initial design x_0 the optimizer searches for a better design by assigning points in the design variable space x_k in which the values of the constraint and objective functions $F_j(x_k)$, $j = 0, \dots, M$ are to be evaluated by response analysis software. Based on the obtained function values a new design point is assigned and submitted for the evaluation to the response analyses software. The process is iteratively repeated until some convergence criteria have been satisfied. The optimization software IMOPT is using an external coupling with response analysis programs. The advantage of such a coupling is that various response analysis software can be used to obtain the values of the constraint and objective functions.

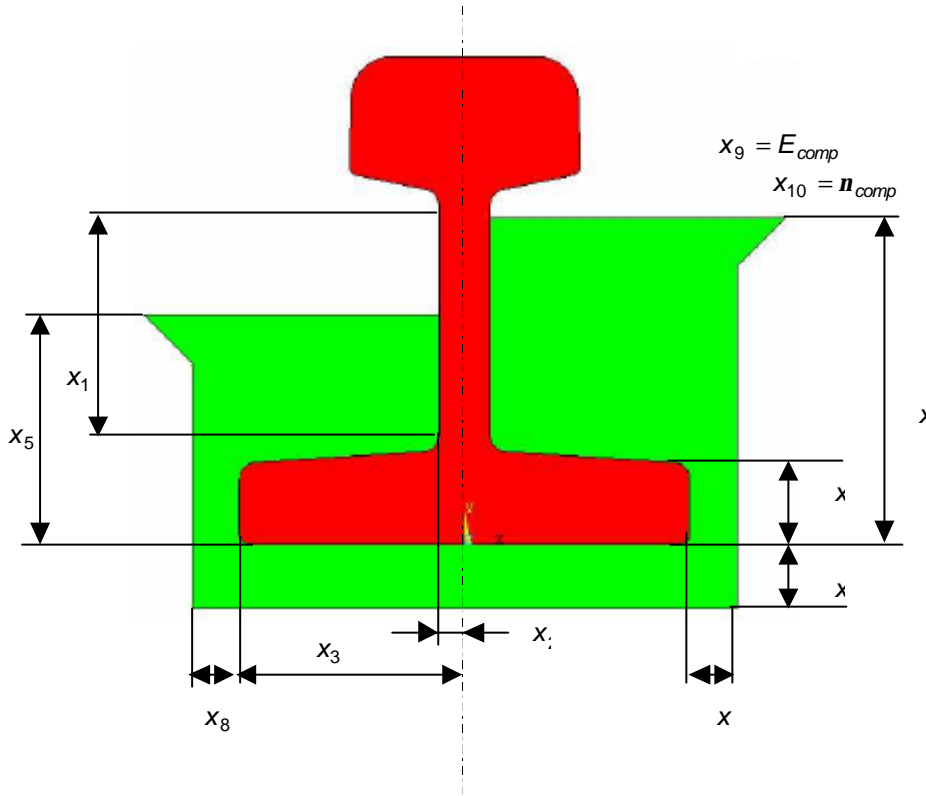


Figure 7 Design variables of the optimization problem

Thus the optimization software is problem independent, i.e. it can be used for various optimization problems.

The optimization method implemented in the IMOPT software is based on the approximation concepts [1]. The method has been specially developed for optimization of real-life problems wherein the objective and constraint functions are computationally expensive. The method has been successfully applied to various static and dynamic problems [see e.g. 10]. For details and most recent developments of the method we refer to [10].

OPTIMIZATION OF ERS

As mentioned above, to use a numerical optimization method a problem should be stated in the form (10) - (12). To optimize ERS, eight geometry parameters and two material parameters of compound have been chosen as the design variables \mathbf{x} . They are shown in Figure 7. Note, that the variables x_5 and x_6 represent the filling level of compound relative to the total height of the rail. The lower and upper bounds of the design variables are collected in Table 1.

Design variable	Lower bound	Upper bound	Initial value	Units
x_1	8.9	85.0	80.0	mm
x_2	8.0	36.0	8.0	mm
x_3	36.0	70.0	70.0	mm
x_4	15.0	30.0	25.0	mm
x_5	0.100	0.750	0.471	-
x_6	0.100	0.900	0.669	-
x_7	5.0	15.0	10.0	mm
x_8	15.0	60.0	15.0	mm
x_9	1.0	10.0	5.6	MPa
x_{10}	0.40	0.46	0.46	-

Table 1 Side limits and initial values of design parameters

For an optimum design of ERS the cost, acoustic noise and maintenance (wheel-rail wear) estimated by the amount of compound, by the area of open surface and resonant frequency (8), and by the contact forces respectively should be minimal, i.e.

$$F_C \equiv V \rightarrow \min, \quad F_N \rightarrow \min, \quad F_M \equiv W \rightarrow \min. \quad (13)$$

The requirements preventing damage of ERS, train derailment (2) and buckling of rails (6) have been used as the constraints:

$$F_1(\mathbf{x}) \equiv \frac{U_{x,2}}{U_{x,allow}} \leq 1, \quad F_2(\mathbf{x}) \equiv \frac{\mathbf{s}}{\mathbf{s}_{allow}} \leq 1 \quad \text{and} \quad F_3(\mathbf{x}) \equiv \frac{P_{bckl}^*}{P_{bckl}} \leq 1 \quad (14)$$

Thus, the optimization problem (13)-(14) has three objectives. Such problems are characterized by an objective conflict, i.e. when improving the values of one objective at least one other objective deteriorates. A typical approach to solve a multi objective problem is to convert it to a single objective one [1]. One way is to consider the most important function as the only objective and to impose limits on the other objective functions (to treat them as the constraints) [3].

The other way is to make a composition of the objective functions while assigning weights (preferences) for each function that reflect the relative importance of the objectives. The solution of the composition problem is a compromise solution since it depends on the weight factors. The whole set of the compromised solutions is called the Pareto set. In practice, a representative subset of the Pareto set is to be determined so that a person in charge (a decision maker) can choose a best appropriate solution [13]. The latter approach has been used here to solve the optimization problem (13)-(14). Thus, the following objective function has been constructed

$$F_0 \equiv \left\{ w_C \frac{F_C}{F_{Cut}} + w_N \frac{F_N}{F_{Nut}} + w_M \frac{F_M}{F_{Mut}} \right\} \rightarrow \min, \quad (15)$$

where

w_C, w_N and w_M are the weight coefficients ($w_C + w_N + w_M = 1$) reflecting the relative importance of compound, noise and maintenance reduction in a final design of ERS;

F_{Cut}, F_{Nut} and F_{Mut} are the normalizing coefficients since the objective functions have different dimensions. The normalizing coefficients are obtained as the solutions (objectives) of the corresponding single criterion optimization problems from (13), the so-called *utopian* solutions since they could never be obtained in the multi criteria optimization.

Each single criterion optimization as well as the final multi criteria optimization have been performed using the MARS method.

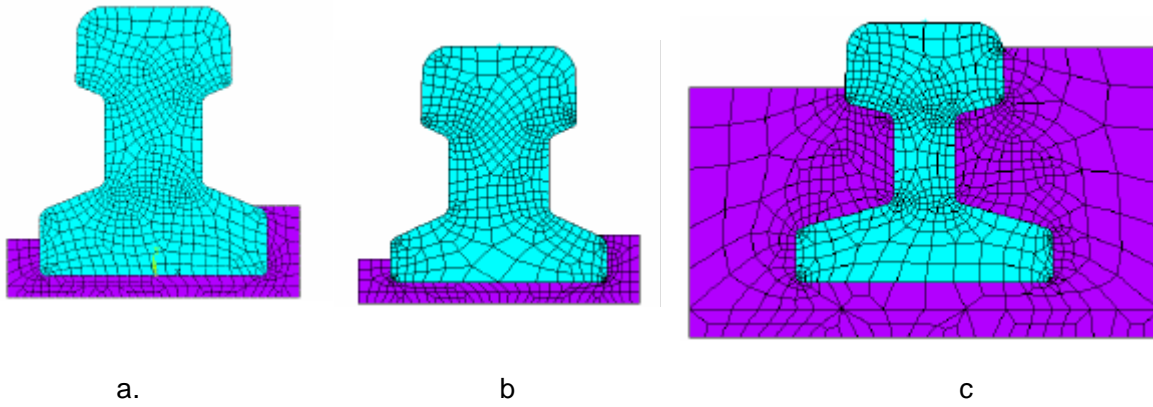


Figure 8 Results of single criterion optimizations (a - compound, b - noise, c - maintenance)

The initial design is shown in Figure 9a and the results of the single criterion optimizations are given in Figure 8. The numerical results are collected in Table 2. From these results it can be seen that the optimum design of single optimization with respect of the cost and noise (Figure 8a and Figure 8b) are more close to each other as to compare with the result of the maintenance optimization (Figure 8c). This can be explained by the fact that compound and noise objectives are not very much conflicting. Both designs have quite good acoustic properties (improved response frequency 350 Hz and 380 Hz respectively and almost a double reduction of the open area A) while they are poor with respect to maintenance properties (high contact forces 14.1 kN and 15.1 kN).

During the third single criterion optimization the properties related to maintenance of ERS have been improved dramatically (contact forces of 17.5 kN for the initial design and 10.0 for the optimized one). However, the acoustic properties are much worse (noise objective 1.09) than the ones of the optimal designs of the two first single optimizations (noise objective 0.54 and 0.46 respectively).

	Initial design	Compound optimization	Noise optimization	Maintenance optimization	Wc=0.33, Wn=0.33, Wm=0.33	Wc=0.1, Wn=0.1, Wm=0.8
Design variables						
x_1 [mm]	0.080	0.041	0.035	0.039	0.041	0.036
x_2 [mm]	0.008	0.022	0.023	0.014	0.022	0.027
x_3 [mm]	0.070	0.053	0.050	0.059	0.053	0.051
x_4 [mm]	0.025	0.030	0.019	0.024	0.030	0.018
x_5	0.471	0.137	0.100	0.749	0.140	0.102
x_6	0.669	0.257	0.200	0.900	0.261	0.201
x_7 [mm]	0.010	0.005	0.005	0.013	0.005	0.005
x_8 [mm]	0.015	0.015	0.015	0.048	0.015	0.016
x_9 [Pa]	5599990	9440470	9988728	4126215	6864797	5386603
x_{10}	0.46	0.43	0.49	0.41	0.46	0.44
Constraints						
s [Pa]	1071470	1145687	2238051	908413	1144731	1086014
$u_{x,2}$ [mm]	1.83	1.08	1.38	1.61	1.44	1.72
P_{bckl} [MN]	12.5	1.7	1.1	5.4	1.7	1.2
Objectives						
F_C [dm ³ /m]	13.536	2.103	1.806	19.973	2.126	1.911
A [dm ² /m]	134.8	74.3	67.5	120.7	74.9	67.5
f_r [Hz]	250	350	380	190	310	290
F_N	1.12	0.54	0.46	1.09	0.57	0.53
F_M [kN]	17.5	14.1	15.1	10.0	12.3	11.2

Table 2 Numerical results of optimizations

Moreover, the amount of compound in the optimum design of the maintenance optimization has become even larger (19.973 dm³/m) than the one in the initial design (13.536 dm³/m).

The small amount of compound above the rail (Figure 8a and Figure 8b) was enough to prevent upward buckling since the stiffness of the rail, which is important for determination of the buckling force according to the chosen analytical criteria (5)-(6), has increased. More accurate calculation of the buckling force using nonlinear numerical analysis might be useful in future optimization.

Two multi criteria optimization problems with different sets of the weight coefficients have been solved. In the first optimization it is assumed that all the objectives are equally important. The optimum design is shown in Figure 9b. In the second problem the reduction of the maintenance effort is more important than the other two objectives. The shape of the optimum design is shown in Figure 9c. The numerical results of these two problems given in Table 2 clearly reflect the effect of the chosen preferences for the optimum design. Other compromised solutions can be found by assigning different preference factors for the objectives and performing a similar optimization. The preference factors can reflect different points of view on optimum design of ERS (point of view of society, investor, maintenance contractor etc). Based on results of the optimizations a decision about a final design can be made.

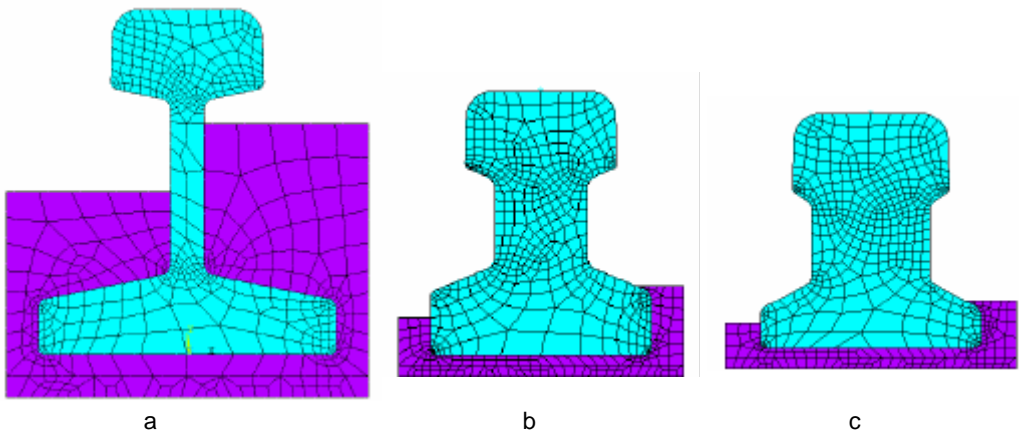


Figure 9 Initial design (a) and results of multicriteria optimization with different weights (b - $w_C = w_N = w_M = 0.33$, c - $w_C = 0.1, w_N = 0.1, w_M = 0.8$)

CONCLUSIONS

A procedure for design of railway track structures using numerical techniques has been presented. The procedure has been applied to optimal design of an Embedded Rail Structure. Both static and dynamic behaviour of ERS have been analyzed using finite element methods, while the optimum search has been directed by an advanced optimization technique.

The multi objective optimization problem has been solved by transforming it to a single objective one. The single objective is composed from the original using weight factors (preferences) that reflect relative importance of the objectives. To prevent numerical difficulties the value of the objective functions have been normalized using the results of the single optimizations (utopian solutions) that have been performed beforehand.

Two optimal designs of ERS have been obtained using different sets of the preferences coefficients. Other solutions can be obtained by performing the optimization with different preferences for the optimum design, which help to make a decision about the final design.

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