A PROCEDURE FOR DESIGN AND OPTIMIZATION OF A RAILWAY TRACK STRUCTURE

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Abstract

Traditionally, railway track design used to be conservative and thus only small changes have taken place in the last century. In the same time, many measurement, analysis and optimization techniques have been developed to understand and predict structural behaviour. And some of these techniques can be introduced to track design as well.

In this paper, a procedure for design of a railway track is presented. It contains several steps to develop optimum track structures under various predefined service, cost environmental conditions. Each track structure is characterized by its component dimensions and mechanical properties such as stiffness and damping. These static and especially dynamic data can be measured in advance or verified afterwards by means of recently developed excitation testing method, which enables manufacturers as well as railway officials to verify track dynamic requirements. This testing method is subject of a study in the current Dynatrack project at Delft University of Technology.

The mechanical behaviour of a track is analyzed using 2-D and 3-D finite element models wherein the track and moving train have been incorporated. The 2-D numerical model has been implemented in the RAIL program. The analysis of the 3-D model can be performed using the general purpose finite element package ANSYS or a dedicated package like CWERRI. Finally, optimum track parameters are determined by applying a numerical optimization technique. The optimization method used here is based on a multipoint approximation approach. The technique has been implemented in a software package IMOPT.

To demonstrate the robustness of the procedure it has been applied to a problem of optimum design of an innovative railway track, a so-called Embedded Rail Structure (ERS). Requirements for the optimum design are related to the wear of rails and wheels, the level of acoustic noise produced by a moving train and the strength of the applied materials. To obtain the optimum design, component dimensions and mechanical properties of the track model have been varied. The results of the optimization are presented and discussed.

Introduction

For a long time design of railway tracks has been a matter of learning from experiences. New insights and new techniques are now able to change this way of doing into a more sophisticated approach, which allows to come to a balanced and even optimum track design.

It is evident that the design of a track structure on the drawing board and in models has to comply, in the best possible way, with the situation in which this structure will be used in the field. The variety of track structures is very large as the field situations are rather different:

Figure 1 TGV and tramway require different types of track

- Different axle loads
- Different operating speeds
- Curved and straight lines
- Urban and rural areas
- Tracks in transitions
- Tracks in level crossings
- Tracks on bridges, in tunnels or in another way integrated in major engineering structures
- Tracks on specific types of soil

The number of types of track structures which are used nowadays, is mainly reduced for reasons of:

- constructability
- maintainability
- reliability

This deals with the fact that standardization in track design and track works reduces costs and possible mistakes. Moreover tracks are generally designed and built with a large reserve in order to avoid a possible failure during operation, or to meet new operational needs in the future. Most track structures, however, are still based on experiences and empirical relations instead of a fundamental study of track behaviour using numerical simulation and optimization techniques.

Track design

One way to perform a study focussing on the track behaviour is outlined below, first in theory and later by means of an example. First two steps in studying the behaviour are schematizing the structure and collecting the necessary parameter data from field or laboratory experiments. These two steps are closely related as schematized structures cannot contain more than the available data, while in some of the advanced schematized structures only a part of the available data will be used.

In order to reduce the complexity of the track design, only the vertical behaviour in straight track is considered in the numerical model. Another assumption is linear elastic behaviour of the applied materials. To perform a full track simulation (which will be discussed in the one-but-next section) the following data is needed:

- The vertical track stiffness, comprising stiffnesses of all flexible components like railpads, elastic compounds, ballast, etc. expressed in:
    - (Quasi-) static values
    - Dynamic values
    - Load-dependent values

Figure 2 Different types of track support

    - Temperature dependent values
    - Life-service dependent values
    - Any combination needed for most realistic calculations.
- Vertical damping values or loss-factors of railpads, ballast, etc.
- Type of support system, e.g. block, sleeper or slab
- Bending and shear stiffness of beam elements such as rails, sleepers or slabs
- Geometry and masses of components such as rails, sleepers or slabs
- Loading situations ranged from a simple unit impulse to a moving train
- Properties of the rail surface collected in a geometry file
- Vehicle parameters including the configuration in dimensions, masses, springs, dampers, etc.

Some of these parameters can be achieved by performing widely applied testing methods for track structures. They are therefore described in international standards [1].

One very important parameter is the vertical stiffness of the track structure. The stiffness value depends on loading conditions. Therefore different stiffness values should be determined in specific test methods. These stiffness values and methods are:

- A static stiffness value is determined from the load-displacement diagram. This diagram is obtained by applying a load slowly increasing from zero till the operational static wheel-load value. The stiffness value is important for static or low speed simulations to analyze the structural strength of the track;
- Quasi-static stiffness values are determined for several harmonic loading cases (1-25 Hz). These values (or an average value) may be of interest for high-speed train simulations;
- A dynamic stiffness value of an unloaded track structure is determined using the impulse excitation method. This method gives a frequency response function, indi-

cating resonance frequencies of the tested track (or track sample). Especially the resonant frequency describing the vertical vibration is suitable for deriving the dynamic track stiffness, vibrating mass and damping values, as given in next figure [2,3]. Dynamic stiffness and damping values are used for analysis of free vibrations of the track, which cause acoustic noise and vibration hindrance.

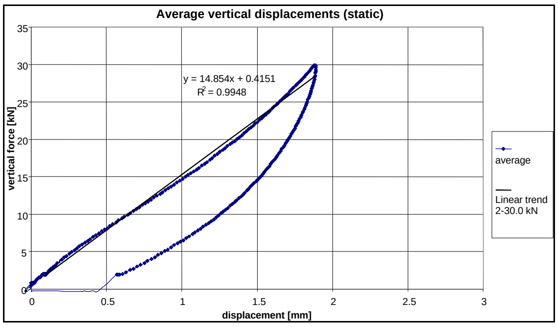

Figure 3 Typical load-displacement diagram of track structure

In some cases, test methods give more data than just a stiffness value. For example creep, hysteresis and damping parameter values can be obtained from the static, quasi-static and dynamic tests respectively. On the other hand test methods should also provide stiffness values as a function of life-service (several millions of loading cycles) or temperature.

Track modelling

For structural analysis of the track, a number of computer packages is available. Especially programs based on finite element method can perform very detailed analysis of displacements, stresses and strains of track components. However such a modelling of a track structure requires a vast amount of elements, especially under loading condition corresponding to a moving train or under other conditions causing wave propagation.

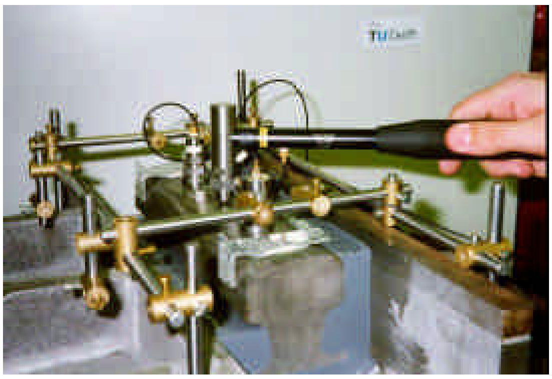

Figure 4 Hammer excitation test on an experimental Embedded Rail Structure

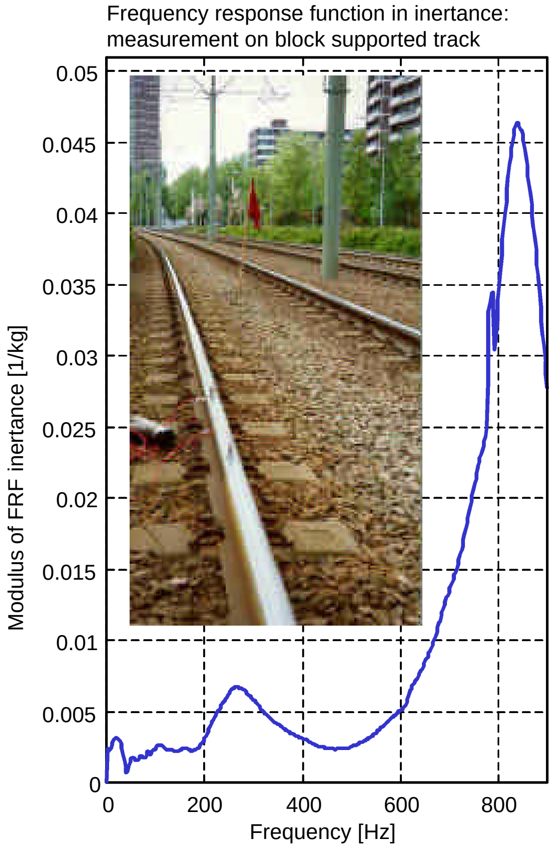

Figure 5 Tested track structure and joining frequency response function

At Delft University of Technology a finite element program for railway track analysis RAIL has been developed [4]. In this program different types of track structures (sleeper, block or continuously supported) are modelled using beam, mass, spring and damping elements (Figure 6). They can be subjected to several types of load:

- static load
- harmonic load
- impulse load
- step load
- static vehicle
- moving vehicle and complete train
- user-defined time dependent load

Other feature of this program is that a whole train can be modelled as a composition of separate bodies, bogies and wheels [5].

In the numerical examples presented here (track analysis and optimization) two loading cases in the RAIL program will be considered, namely the impulse load and the moving vehicle. In these two cases the results of dynamic load test are used.

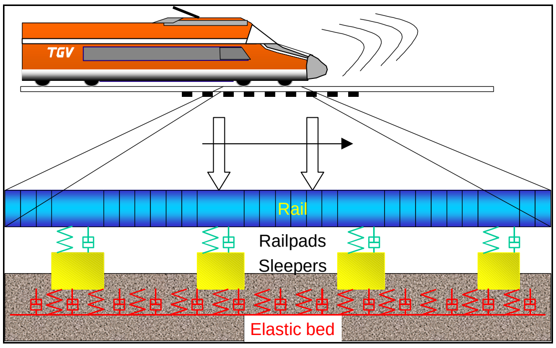

Figure 6 Modelling of vehicle and track structure in RAIL

Apart from the simulations using RAIL program, a structural 3-D analysis of the railway structure on a smaller scale can be performed to analyse e.g. effects caused by geometry or the spatial material behaviour. Based on the results of the analysis such as the stress distribution in the structure, the stress concentrations can be determined. For this purpose any general purpose structural analysis software can be used, e.g. ANSYS. In these simulations the results of static load tests in different situations can be used.

Other track simulations can be performed by CWERRI [6], a finite element package for analysis of stability in longitudinal and lateral directions of rails, tracks and entire engineering structures such as bridges, tunnels, etc.

Track optimization

When the behaviour of the railway track structure has been analyzed, the next step is to optimize it. Optimization of a mechanical structure generally means improving the structure performance during the working cycles under some manufacturing, operational and failure conditions as well as cost limitations. In the traditional design of such structures the optimization is carried out in a primitive way by modification of separate design parameters and repeated numerical analyses. However, it is a time consuming process and, moreover, the success can not be guaranteed. The most systematic way to improve the design is to use numerical optimization techniques. Being coupled with advanced numerical simulation analysis, they search for an optimum design after which possible prototypes can be built. An application of such optimization technique to design of the Embedded Rail Structure is presented below.

General optimization problem

To make a use of numerical optimization techniques the optimization problem should be stated in a general form that reads

Minimize

$$F_0(\boldsymbol{x}) \rightarrow \min \text{ , } \boldsymbol{x} \in \boldsymbol{R}^N \qquad (1)$$

subject to

$$F_j(\boldsymbol{x}) \leq 1, \quad j = 1,\dots,M \qquad (2)$$

and

$$A_i \leq x_i \leq B_i, \quad i = 1,\dots,N \,. \qquad (3)$$

Here

F_0 is *an objective function*;

F_j, $j = 1,\dots,M$ are *constraints*;

$\boldsymbol{x} = (x_1,\dots,x_N)^T$ is a vector of *design variables*,

A_i and B_i are the so-called *side limits*, which define lower and upper bounds of the *i*-th design variable.

The components of the vector $\boldsymbol{x}$ represent various parameters of the mechanical system, such as geometry, material, stiffness and damping properties, which can be varied to improve the performance characteristics of the system. Depending on the problem under consideration the objective and constraint functions (1)-(2) can describe various structural and dynamic response quantities of the system such as weight, reaction forces, stresses, natural frequencies, displacements, velocities, accelerations, *etc*. Cost, maintenance and safety requirements can be used in the formulation of the optimization problem as well. The objective function provides a basis for improvement of the design whereas the constraints impose some limitations on the behaviour characteristics of the system.

Solution of the optimization problem is an iterative process, which involves multiple evaluations of the objective and constraint functions (1)-(2). Typically, the values of the functions can be obtained using one of the numerical methods, e.g. the Finite Element Method.

Approximation concept

The optimization problem (1)-(3) can be solved using a conventional method of mathematical programming. However, for systems with many degrees of freedom the finite element analysis can be time consuming. As a result, the total computational effort of the optimization might become prohibitive. This difficulty has been mitigated in the mid-seventies by introducing approximation concepts [7].

According to the approximation concepts the original functions (1)-(2) are replaced with approximate ones which are computationally less time consuming. Instead of the original optimization problem (1)-(3) a succession of simpler approximated subproblems similar to the original one formulated using the approximation functions is to be solved. Each simplified problem then has the following form:

Minimize

$$\tilde{F}_0^k(\boldsymbol{x}) \to \min,\ \boldsymbol{x} \in \boldsymbol{R}^N \qquad (4)$$

subject to

$$\tilde{F}_j^k(\boldsymbol{x}) \le 1, \quad j = 1,\dots,M \qquad (5)$$

and

$$A_i^k \le x_i \le B_i^k,\ A_i^k \ge A_i,\ B_i^k \le B_i,\ i = 1,\dots,N,\ (6)$$

where the superscript k is the number of the iteration step, $\tilde{F}$ is the approximation of the original function F, A_i^k and B_i^k are *move limits* defining the range of applicability of the approximations.

Since the functions (4)-(5) are chosen to be simple and computationally inexpensive, any conventional method of optimization [7] can be used to solve the problem (4)-(6). The solution of the problem $\boldsymbol{x}_*^k$ is then chosen as starting point for the (k+1)-th step and the optimization problem (4)-(6) re-formulated with new approximation functions $\tilde{F}_j^{k+1}(\boldsymbol{x}) \le 1, \quad (j = 0,\dots,M)$ and move limits A_i^{k+1} and B_i^{k+1} is to be solved. The process is repeated until the convergence criteria are satisfied.

Multipoint approximation method

The approximation is defined as a function of the design variables $\boldsymbol{x}$ and tuning parameters $\boldsymbol{a}$ (for brevity the indices k and j will be omitted). To determine the components of vector $\boldsymbol{a}$ the following weighted least-squares minimization problem is to be solved [8]:

Find vector $\boldsymbol{a}$ that minimizes

$$G(\boldsymbol{a}) = \sum_{p=1}^{P} \{ w_p^{(0)} [F(\boldsymbol{x_p}) - \tilde{F}(\boldsymbol{x}_p, \boldsymbol{a})]^2 \} \qquad (7)$$

Here $F(\boldsymbol{x_p})$ is the value of the original function from (1)-(2) evaluated at the point of the design parameters space $\boldsymbol{x_p}$, and P is the total number of such points; $w_p^{(0)}$ is a weight factor that characterises the relative contribution of the information about the original function at the point $\boldsymbol{x_p}$. For the numerical examples the multiplicative form of the approximating function has been chosen, which has the form

$$\tilde{F}(\boldsymbol{x}) = a_0 \prod_{i=1}^{P} (x_i)^{a_i} \qquad (8)$$

The optimization process is controlled by changing the move limits in each iteration step. The main rules of the strategy of changing of the move limits employed in the method are:

- if the approximating functions do not adequately represent the original ones in the current optimum point, what means that the search subregion is larger than the range of applicability of the current approximations, the move limits (6) are changed to reduce the size of the search subregion;
- if the approximations are good and the solution of the optimization problem (4)-(6) is an internal point of the search subregion it could be considered as the solution of the original optimization problem (1)-(3), the search subregion is reduced;
- if the current optimum point belongs to the boundary of the search subregion (one of the move limits is active) whereas the approximations are good the size of the subregion is not changed on the next iteration.

The iteration process is terminated if the approximations are good, none of the move limits is active and the search subregion is small enough. More details about the weight coefficients assignment, the move limits strategy and the most recent developments of the method can be found in [9].

Objectives for track optimization

Formulating the objectives for optimized track structures requires first a keen consideration on which response quantities characterize the performance of track structures. The following objectives which are considered to be important, have been selected for optimization:

Objective 1. Based on the frequency response function (FRF), resonant frequencies of the track structure should coincide neither with vehicle resonant frequencies, nor with other structural resonant frequencies. Amplification at these frequencies should be restricted [10]. These conditions lead to better riding characteristics of the vehicle and less effort on maintenance of tracks, vehicles and structures.

Objective 2. The acoustic characteristics of the track structure can be estimated by a specific response quantity, a so-called attenuation rate (distance damping). The attenuation rate characterizes the ability of a track structure to damp the vibrations on different distances from the source. Acoustic noise radiating from the rails decreases as the attenuation rates increase. And this is favourable for the direct environment of railway lines.

Objective 3. Wheel-rail contact forces should be below prescribed values in order to reduce rail and wheel wear. These forces strongly depend on the rail surface geometry. The standard deviation of these forces should be lower than e.g. 20% of the static wheel-rail contact force [11]. This percentage is based on simulations with a moving vehicle on an average quality rail surface geometry for a sufficient time period.

Apart from the above mentioned objectives, other could be formulated and taken into account during optimization.

Example

In this Section the proposed approach is demonstrated by applying it to the design and optimization of a specific slabtrack system, a so-called Embedded Rail Structure (ERS). Netherlands Railways has applied this system since the 60's on engineering structures (bridges) and since the early 70's in concrete slabs as well as in level crossings. Later, ERS has found its way to other companies, including tramway and metro, all over the world.

Figure 7 Embedded Rail Structure installation near Best (the Netherlands)

Recently a 3km slabtrack with ERS has been installed in the southern part of the Netherlands (near Best). This structure consists of a reinforced continuous concrete slab on a concrete stabilized roadbed, which is placed on a sand base. In the slab two troughs at 1.5m spacing serve to incorporate UIC54 rails, a visco-elastic compound called Corkelast®, an elastic strip at the bottom of the troughs and some construction utensils.

The above described ERS will be used later for the optimal design.

Existing track structure: testing and simulation

Geometry and product data came from Netherlands Railways (NS Railinfrabeheer) and Edilon. Two important model parameters have been determined based on the excitation hammer test. Dynamic stiffness and damping have been determined by performing this test with a 0.5 m sample of ERS. Five consecutive measurements in the unloaded situation yielded results for the vertical vibration behaviour of the rail in the visco-elastic compound as shown in Figure 9.

The stiffness (k) and damping (c) parameters of this compound have been obtained by fitting the FRF of a single degree of freedom mass-spring-damper system into the experimental data.

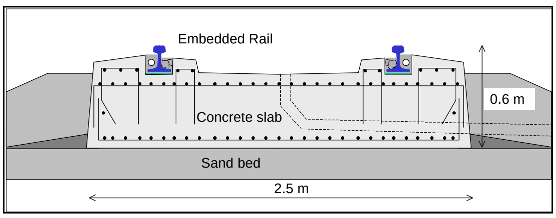

Figure 8 Schematic cross-section and detail of ERS

Frequency response function

An FRF has been computed for an infinite ERS, which was for calculation purposes limited to 40m with elements of 0.04 m long. Compared to the simulated FRF, an in-situ test has been carried out on the ERS in the field. The results are depicted in Figure 10. The slight differences in the results near the resonant frequencies (170 Hz simulation vs. 160 Hz field test) are due to accuracy reasons in the analysis and due to unspecified field conditions.

The FRF's of the ERS clearly show the vertical vibration preference of the rail in the trough. The vibration of the slab on the concrete stabilized roadbed occurs at considerable low frequencies, not discoverable with the testing equipment, but visible in simulation results around 20 Hz.

As the amplification at resonant frequency of the structure is high, damping is relatively low. This can be interest aspect to be considered for further optimization.

Attenuation rates

A second type of simulation gives attenuation rate results, which are in fact a decay rate values of frequency response function evaluated at fixed distances from the excitation point. These points are located at 0.5 1.0, 2.0, 4.0, 8.0 and 16.0 m. All absolute differences are first calculated in dB, then divided by their distance and finally averaged.

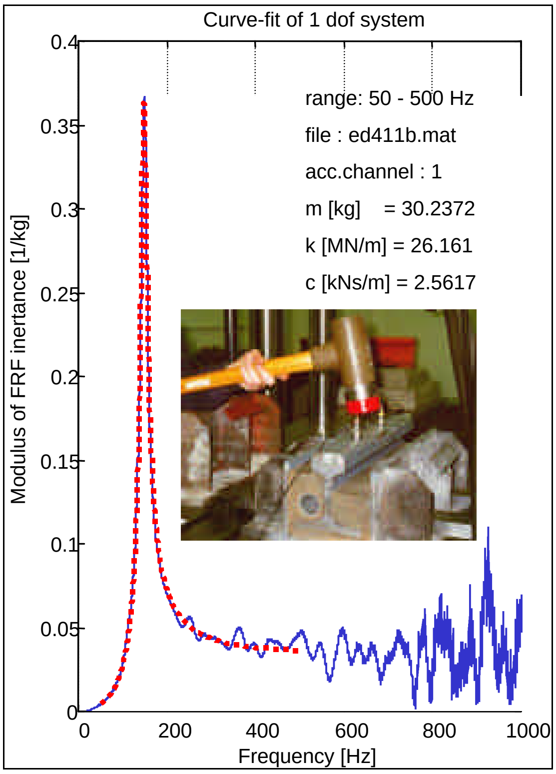

Figure 9 FRF of a 0.5 m sample of ERS

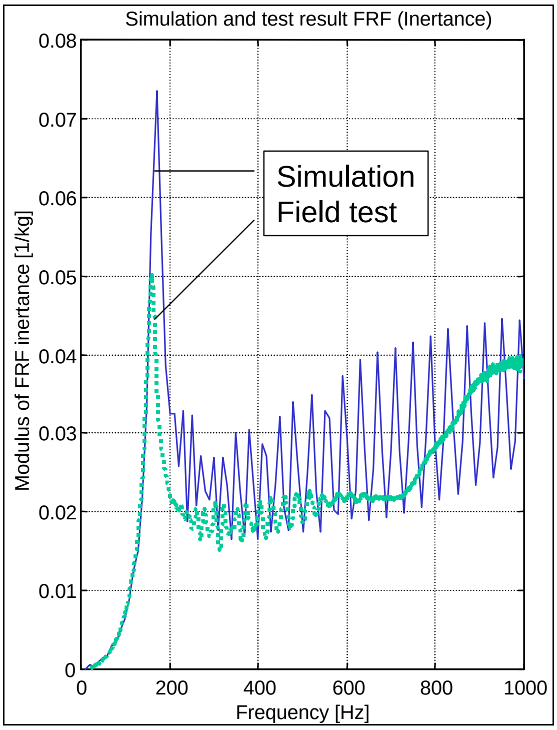

Figure 10 Simulation and field test results of impulse loading on ERS

For the ERS (Table 2) with the UIC54 rails the simulation results in a maximum attenuation rate of approximately -4 dB/m in the range below the resonant frequency of 170 Hz.

Wheel-rail contact forces

For this simulation a file with vertical rail profile data has been created. It contains waves, which lengths range from 5 cm to 3.0 m. The amplitude of each wave is evaluated according to the empirical and simplified formula:

$$A = 0.0001\, s\sqrt{l}\ , \qquad (9)$$

where

l [mm] is the wavelength;
s is a scaling coefficient.

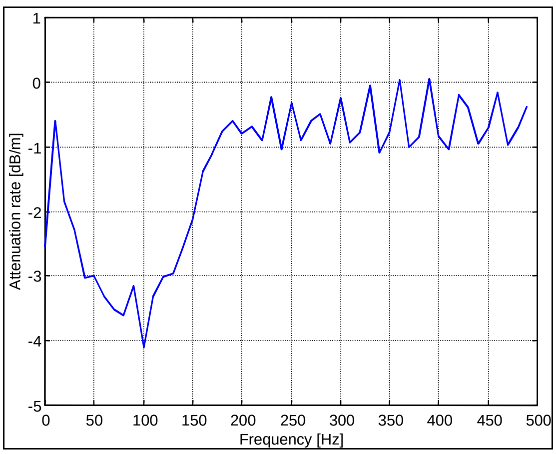

Figure 11 Attenuation rate of ERS

The waves are combined with random phase-difference. The standard deviation of this profile is approximately 0.5 mm (for s=1.0) that corresponds to high-speed lines of average quality. The rail (total length 270 meters) is modelled using elements of 0.01 m.

Results of simulations at 3 different velocities (30, 60 and 90 m/s) are compared on maximum and average wheel-rail contact forces which corresponds to one wheel of TGV motorcar. Each simulation takes 6/90 seconds, that corresponds the track length of 2, 4 or 6 m respectively. The standard deviations of the wheel-rail force on this full distance (including possible initial effects) are collected in Table 1.

	30 m/s	60 m/s	90 m/s	Unit
St.dev. (s)	12.05	28.59	38.84	kN
Static wheel load (μ)	85	85	85	kN
Ratio (σ/μ)	14.2	33.6	46.0	%

Table 1 Wheel-rail contact forces (initial design of ERS)

This table shows that deviation in wheel-rail forces increases with velocity according to an almost linear extend.

Optimization of ERS

Using the requirements to the Embedded Rail Structure formulated in the previous section, two optimization problems have been formulated. These problems have been solved for three different velocities of the moving train. For the model of the train the realistic data of the TGV train has been used. Both problems have been solved using the optimization method described above, whereas the multiplicative function (8) has been used to approximate the objective and constrain functions. The optimization problems and results are presented and discussed below.

Single criterion optimization of ERS

In this problem the requirements related to the acoustic noise hinder and wheel-rail wear (see previous Section) have been chosen for the optimum design of the ERS. To formulate the optimization problem in the general form (1)-(3) the first requirement has been taken as the criterion for the optimization (which defines the objective function). The other requirement has been considered as a constraint. Thus, the following optimization problem is to be solved:

For a given train moving with a prescribed velocity along the ERS track with a given rail surface profile minimize the inverse resonant frequency of the structure f_0

$$F_0(\mathbf{x}) = \frac{1}{f_0} \quad \rightarrow \quad \min . \qquad (10)$$

The constrain has been imposed on the standard deviation of the wheel-rail contact force $s\,(P)$, which should not exceed 20% of its static value P_{st} that reads

$$F_1(\mathbf{x}) = \frac{s\,(P)}{P_{st}} \le 0.2 . \qquad (11)$$

The resonant frequency defines the level of the acoustic noise produced by a moving train, whereas the contact force describes the wheel-rail wear. To obtain the optimum design of the ERS the stiffness (k) and damping (c) parameters of the visco-elastic compound have been chosen as design variables. Their lower and upper bounds are given in Table 2.

	Initial value	Lower bound	Upper bound	Unit
k	52	10	100	MN/m/m'
c	5.2	1	15	kNs/m/m'
s	0.5	0.1	1.5	-

Table 2 Design variables

The optimization problem has been solved for the three train velocities. The numerical results of the optimization are given in Table 3, the attenuation rates and contact forces are shown in Figure 12 and Figure 13 respectively.
The constraints were satisfied for all three optimum designs ($s = 17.8\,kN$). From Table 3 it can be seen that the optimum values of the stiffness and damping parameters of the compound decrease as the velocity of the

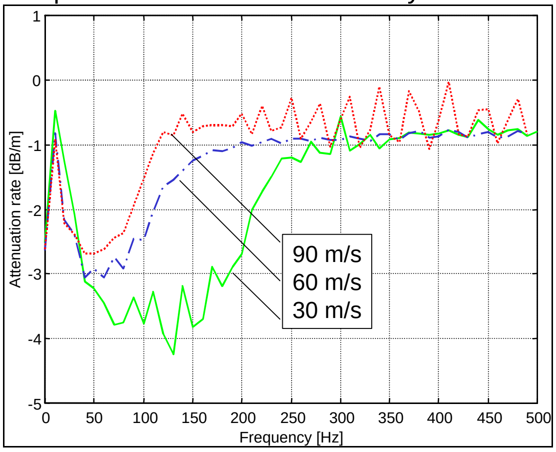

Figure 12 Attenuation rate of optimum designs (single criterion optimization)

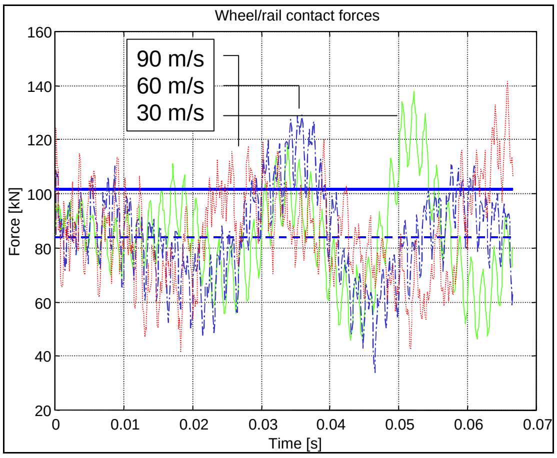

Figure 13 Wheel-rail contact force of optimum designs (single criterion optimization)

train increases. Comparing the attenuation rate results it can be concluded that the optimum design of ERS for a low-speed train has better acoustic properties that the ERS for a high-speed train.

Multicriteria optimization of ERS

In this problem the optimization searches for an optimum design of ERS track that requires minimum maintenance effort and that has good acoustic properties. The maintenance effort can be estimated by the degree of roughness of the rail surface while the acoustic properties can be described by the resonant frequency of the ERS structure.

To combine these two criteria the following objective function has been used

$$F_0(\mathbf{x}) = \frac{f_{\max}}{f_0} + \frac{s_{\max}}{s} \quad \rightarrow \quad \min\,, \qquad (12)$$

where

s - is the scaling coefficient defining the roughness of the rail surface (see (9));

$f_{\max}$ and $s_{\max}$ - are the maximum values of f_0 and s respectively.

The optimum design should satisfy the optimum wheel-rail wear requirement as well, i.e. the standard deviation of the contact force should be equal to the prescribed level (20% of static load). The vector of the design variables $\mathbf{x}$ comprises the stiffness and damping parameters of the compound, and the scaling coefficient s. The lower and upper bounds as well as the initial values of the design variables are given in Table 2. The train parameters and velocity have not been changed during the optimization.

Again, three optimization problems using different velocities of the train have been solved. The results are given in Table 4, Figure 14 and Figure 15. From the Table it can be seen that the damping in the first two optimizations has reach its upper bound whereas in the third problem the stiffness is equal to the lower limit (Table 2 and Table 4). That explains the fact that the optimum value of the scaling coefficient is larger in the optimization for 90m/s then in the one for 60 m/s. To prevent this, a critical damping indicating energy absorption of the structure

	30 m/s	60 m/s	90 m/s	Unit
Optimized stiffness k	88.8	28.4	20.9	MN/ m/m'
Optimized damping c	12.5	13.8	6.46	kNs/ m/m'
Resonant frequency	200	110	100	Hz
Attenuation rate (max.)	-4.2	-3.0	-2.7	dB/m
Scale of surface profile	0.5	0.5	0.5	-
St.dev.contact forces (s)	17.8	17.8	17.8	kN
Static wheel load(μ)	85	85	85	kN
Ratio (σ/μ)	20	20	20	%

Table 3 Results of single criterion optimization of ERS (visco-elastic compound)

	30 m/s	60 m/s	90 m/s	Unit
Optimized stiffness k	39.5	21.1	10.0	MN/ m/m'
Optimized damping c	15.0	15.0	10.8	kNs/ m/m'
Resonant frequency	140	100	60	Hz
Attenuation rate (max.)	-3.4	-2.7	-2.8	dB/m
Optimized scale coef. (s)	1.11	0.61	0.71	-
St.dev.contact forces (s)	17.8	17.8	17.8	kN
Static wheel load (μ)	85	85	85	kN
Ratio (σ/μ)	20	20	20	%

Table 4 Results of multicriteria optimization of ERS (visco-elastic compound AND rail surface geometry)

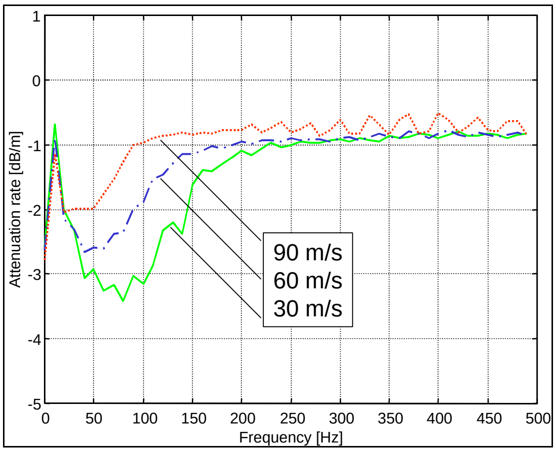

Figure 14 Attenuation rate of optimum designs (multicriteria optimization)

during long time repetitive loading (fatigue test) should be taken into account and additional constraint should be imposed on the value of the critical damping ratio.

Conclusions

A procedure for design of railway track structures, which includes numerical modelling and dynamic analysis along with laboratory testing and optimization, has been presented.

The computer program RAIL for the analysis of railway structures under (high-speed) train loading has been described. The numerical model was verified using an excitation hammer test.

The procedure has been applied to the design of an Embedded Rail Structure for various train velocities.

Several criteria for the optimization of an ERS have been proposed. Mechanical properties of the ERS have been determined using single and multiple criteria. The applied criteria have been related to the acoustic

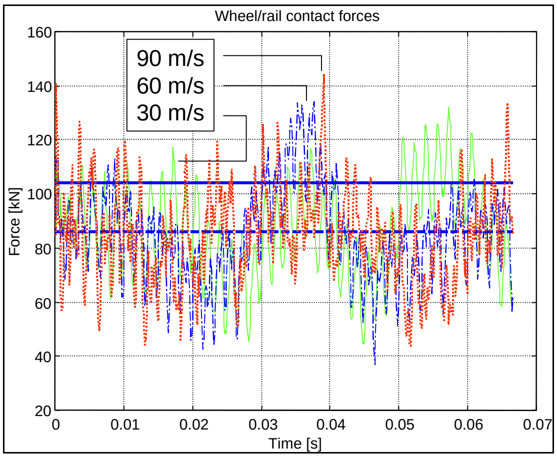

Figure 15 Wheel-rail contact force of optimum designs (multicriteria optimization)

noise and vibration performance, as well as track maintenance requirements. The problems have been solved using a numerical optimization technique.

The results showed that the proposed procedure can effectively be used for the design of railway structures.

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