OPTIMIZATION OF THE DYNAMIC RESPONSE OF LINEAR MECHANICAL SYSTEMS USING A MULTIPOINT APPROXIMATION TECHNIQUE*

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ABSTRACT

The optimization problem of the ride characteristics of a travelling truck is considered. Its dynamic behaviour is approximated by linear FE models (both 2-D and 3-D). The road surface profile is presented as a random function with known power spectral density. The design variables comprise geometry as well as spring and damper properties. Limitations are imposed on maximum values of the relative displacements of suspensions, dynamic wheel load to axles, acceleration of the cargo.

The above problem is solved using a multipoint approximation method. To reduce the computational cost a two-level optimization procedure for the truck optimization problem is proposed.

It is demonstrated that the method used is efficient for optimizing the dynamic behaviour of complex structures and it is also promising for geometrically non-linear dynamic problems. Moreover it can easily be coupled with a general-purpose finite-element software package.

1. Introduction

Approximation concepts are now very popular in practical optimization [1]. In most cases the optimization process involves many calculations of objective and constraint functions and/or their derivatives, this often implies use of some numerical response analysis technique. Considering the design of a large engineering system such as a ground vehicle, the response analysis can be time consuming. The computational effort is considerably reduced by introducing approximation concepts, the original functions are then replaced by simplified and explicit ones.

Sometimes, optimization is difficult even if one of the approximation methods is employed because of a large number of design variables and constraints. A most common solution is to break the problem into several smaller subproblems and a coordination problem. Several procedures for decomposing the large structural optimization problem into subproblems have been proposed in the literature (e.g. [2]).

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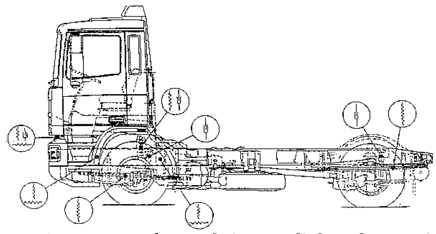

Figure 1. Suspension system of a truck (I.Besselink and F.van Asperen, [3])

For a problem which combines sizing (stiffness and damping properties of elements) and geometry (nodal coordinates) optimization, a typical decomposition is to consider the geometrical variables as the upper-level (global) variables and the sizing variables as the lower-level (local) ones [7]. This is motivated by the fact that the geometrical and sizing design variables are of fundamentally different nature which can lead to numerical difficulties if they are treated together in a single-level optimization [8].

In the present paper a problem of optimization of ride characteristics of a truck is considered. The ride characteristics are related to the vibration of the vehicle due to road irregularities and its effects on a driver and goods. The dynamic behaviour of the truck is described by a linear finite element model and a road surface profile is presented as a random function with known power spectral density. The comfort of the driver is estimated by means of a ride index calculated as a weighted mean square acceleration at the point of the driver's seat. To minimize the ride index, stiffness and damping coefficients of suspension elements and coordinates of several nodes are varied with limitations imposed on the RMS value of the responses such as the relative displacements of the suspensions, dynamic wheel load of the axles, acceleration of the cargo. This problem is solved using a multipoint approximation (MPA) technique [11].

To reduce computational efforts a two-level procedure was developed. Three sizing sub-optimization problems related to the axles, cabin and engine subsystems are considered at the lower level, and the optimization of the geometry at the top level. The results of the optimization of the 2-D and 3-D finite element model of the truck are presented and discussed.

2. Design Problem

2.1. CALCULATION OF DYNAMIC RESPONSE

A truck moving on the road represents a complex vibratory system. It contains the truck and trailer masses, connected to a chassis by suspensions, and resting on axles and tires, while the road represents the excitation input applied to the tires (Figure 1). The dynamic response of such a system depends on the characteristics of the vehicle elements as well as on the road quality.

Assuming only relatively small displacements, the dynamic behaviour of a vehicle can be described by a linear model. The simplified 2-D finite element model of the truck is shown in Figure 2. The cabin and engine are modelled as rigid bodies, beam elements are used for the chassis, whereas linear spring and damping elements describe the cabin and axle suspensions and the engine mountings.

In a simple case, the excitation inputs caused by the road roughness can be described by periodic functions. But, in order to obtain more realistic results the road surface profile should be regarded as a random function with known power spectral density (PSD) which is different for the various types of roads ranging from an unprepared terrain to a highway. In the present study the dynamic response of the truck was evaluated for a "standard" road profile with PSD approximated by the function $S_p(f) = 10^{-6}V/f^2 \quad [m^2/Hz]$, where V is the speed of the truck [*m/s*], f is the frequency [*Hz*].

For a linear mechanical system, a direct linear relationship between inputs and output exists [10]. If a system has R excitation inputs it reads

$$S_q(f) = \sum_{r=1}^{R}\sum_{s=1}^{R} H^*_{p_r}(f)\, H_{p_s}(f)\, S_{p_r p_s}(f) \qquad (1)$$

where S_q - response PSD; H - complex frequency response function; H^* - complex conjugate of H; $S_{p_r p_r} = S_{p_r}$ - PSD of input $p_r(t)$; $S_{p_r p_s}$ - power cross-spectral density of inputs $p_r(t)$ and $p_s(t)$. The frequency response function H_{p_r} is calculated as a response of a system to the excitation input p_r assumed to be a unit harmonic excitation, i.e. $p_r(t) = e^{2\pi i f t}$. The harmonic response analyses were fulfilled using the ANSYS finite element package [12]. When the response PSD has been determined, the mean square response can be calculated directly from

$$E[q^2] = \int_0^\infty S_q(f)\, df \qquad (2)$$

The mean square of the velocity and acceleration can be easily expressed through the PSD of the displacement.

The 2-D model of the truck is affected by two excitation inputs applied at the road contact points of the front and rear wheels with the same PSD $S_{p_1} = S_{p_2} = S_p$. Moreover, since the rear wheels encounter the same road irregularities as the frontal ones (after a time delay T), i.e. $p_1(t) = p_2(t+T)$, the corresponding inputs can be considered as perfectly correlated, so that their power cross-spectral densities are $S_{p_1 p_2} = S_p e^{-2\pi i f T}$ and $S_{p_2 p_1} = S_p e^{2\pi i f T}$. The time delay $T = L/V$ depends on the length of the wheelbase L and the speed V of the truck.

To assess the driver's comfort, a specific kind of mean square response - ride index F_0 - is calculated. It is defined as a weighted mean square acceleration and can be written as

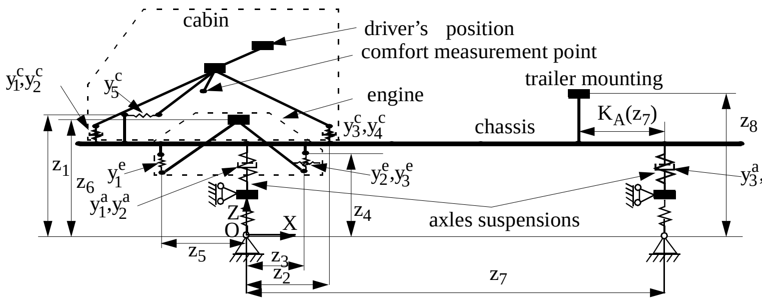

Figure 2. The 2-D FE model of the truck and the design variables

$$F_0 = \left(\int_0^\infty [W^l(f)]^2 S^l_{q''}(f)\,df + \int_0^\infty [W^v(f)]^2 S^v_{q''}(f)\,df + \int_0^\infty [W^{lt}(f)]^2 S^{lt}_{q''}(f)\,df \right)^{1/2} \quad (3)$$

where $S^l_{q''}$, $S^v_{q''}$ and $S^{lt}_{q''}$ are the PSD of the acceleration in the longitudinal, vertical and lateral direction respectively. The weight functions (W^l, W^v, W^{lt}) reflecting the human sensitivity to vibrations in a particular direction are taken from ISO standard 2631 [5].

In the problem under consideration all the responses were restricted to the frequency range from 0 to 20 Hz while the speed of the truck $V = 22.2 m/s$ $(80 km/h)$.

2.2. FORMULATION OF OPTIMIZATION PROBLEM

As it is mentioned above, the road surface irregularities result in vibrations which affect the driver and goods. They can significantly deteriorate the comfort of the former and simply damage the latter. That is why reduction of the undesirable vibrations is of great importance in the design of such a vehicle. The optimization problem was stated as:

Minimize the ride index $F_0(\boldsymbol{x})$ at a comfort measurement point (Figure 2) for a given road profile.

The stiffness and damping of the suspension elements (Figure 1) and the coordinates of several nodes were chosen as components of the vector of design variables $\boldsymbol{x}$. The design variables $\boldsymbol{x} = [\boldsymbol{y}, \boldsymbol{z}]$ shown in Figure 2 can be classified as follows:

$\boldsymbol{y} = [\boldsymbol{y}^a, \boldsymbol{y}^c, \boldsymbol{y}^e]$ is the subvector of sizing design variables;

$\boldsymbol{z}$ is the subvector of geometrical design variables.

The subvectors $\boldsymbol{y}^a, \boldsymbol{y}^c$ and $\boldsymbol{y}^e$ are related to the axle, cabin and engine subsystems respectively. The lower and upper bounds of the design variables are collected in Table 1.

Written in the dimensionless form the constraints can be classified as follows:

$$F_j(\boldsymbol{y}^a, \boldsymbol{z}, [\boldsymbol{y}^c, \boldsymbol{y}^e]) \le 1, \quad j = 1, 2 \quad (4a)$$

$$F_j(\boldsymbol{y}^c, \boldsymbol{z}, [\boldsymbol{y}^a, \boldsymbol{y}^e]) \le 1, \quad j = 3, 4, 5 \quad (4b)$$

$$F_j(\boldsymbol{y}^e, \boldsymbol{z}, [\boldsymbol{y}^a, \boldsymbol{y}^c]) \le 1, \quad j = 6, 7, 8 \quad (4c)$$

$$F_j(\boldsymbol{y}^a, \boldsymbol{z}, [\boldsymbol{y}^c, \boldsymbol{y}^e]) \le 1, \quad j = 9, 10 \quad (4d)$$

$$F_{11}(\boldsymbol{z}, [\boldsymbol{y}^a, \boldsymbol{y}^c, \boldsymbol{y}^e]) \le 1 \quad (4e)$$

Here (4a)-(4c) are the limitations on the RMS value of the relative displacements of the axles and cabin suspensions and the engine mountings respectively, (4d) constraint the maximum value of the dynamic wheel load of the axles and (4e) refers to the RMS acceleration of the cargo. The displacements due to static loads were also taken into account. A more detailed information on the constraints is given in [3, 9].

It should be noted, that the constraint functions depend only weakly on the subvectors of the design variables in brackets. The geometrical design variables affect all the constraints though not so strong as the sizing relevant ones. These facts will be used later for the decomposition.

3. Multipoint approximation technique

The MPA method used in this study is described in detail in [11] and therefore it is only briefly presented here.

According to a general approximation concept the original optimization problem is replaced by a succession of the simpler ones, namely in each iteration step k the following problem is to be solved:

$$\text{Find } \boldsymbol{x}_*^k \text{ such that } \quad \tilde{F}_0^k(\boldsymbol{x}, \boldsymbol{a}_0) \rightarrow min\,, \qquad \boldsymbol{x} \in R^N \tag{5}$$

$$\tilde{F}_j^k(\boldsymbol{x}, \boldsymbol{a}_j) \le 1\,,\ \boldsymbol{a}_m = [a_1^m a_2^m \ldots a_L^m]^T\,, \qquad j = 1, \ldots, M\,, \qquad m = 0, \ldots, M \tag{6}$$

$$A_i^k \le x_i \le B_i^k\,, \qquad A_i^k \ge A_i\,, \qquad B_i^k \le B_i\,, \qquad i = 1, \ldots, N \tag{7}$$

Each functions $\tilde{F}(\boldsymbol{x}, \boldsymbol{a})$ (the indices j and k are suppressed to simplify the notations) is an explicit approximation of the original function $F(\boldsymbol{x})$. A vector of tuning parameters $\boldsymbol{a}$ is determined on the basis of the information about the original function (and possibly its derivatives [11]) at several points of the design space. The evaluation of the components of the vector $\boldsymbol{a}$ is formulated as a weighted least-squares optimization problem:

Find the vector $\boldsymbol{a}$ that minimizes

$$G(\boldsymbol{a}) = \sum_{p=1}^{P} \{ w_p^{(0)} [F(\boldsymbol{x}_p) - \tilde{F}(\boldsymbol{x}_p, \boldsymbol{a})]^2 \} \tag{8}$$

where $w_p^{(0)}$ is a weight coefficient that characterizes the relative contribution of the information about F at the point $\boldsymbol{x}_p$ [11]. In [4] it is shown that a most accurate approximation can be attained using *a multiplicative* function

$$\tilde{F}(\boldsymbol{x}, \boldsymbol{a}) = a_0 \prod_{l=1}^{N} x_l^{a_l} \tag{9}$$

The minimum number of the points P needed to determine the coefficients $\boldsymbol{a}$ is equal to $N + 1$. They are obtained by taking steps from the starting point (optimum from the previous iteration) in each coordinate direction of the design space. The quality of the approximation can be improved without additional effort by taking the points from the previous steps (belonging to the current subregion and its neighbourhood) into account.

The strategy of changing of the move limits can be summarized as follows. After each iteration step k the search subregion defined by the move limits A^k and B^k is reduced if in the obtained point $\boldsymbol{x}_*^k$: 1) the approximation is not adequate at least for one active constraint or 2) none of the move limits is active (the obtained point is internal). If the above conditions are both satisfied the same move limits are used in the next iteration step [11].

The optimization process is terminated when

- the approximations are adequate for all active constraint in the obtained point
- the obtained point is internal
- the subregion has reached a prescribed small size: $max\left[\dfrac{B_i^k - A_i^k}{B_i - A_i}\right] \le 0.01$, i=1, ..., N.

4. Two-level solution

The considered optimization problem can directly be solved using the multipoint approximation method. However, the computational cost considerably increases if a more complex 3-D finite element model is used in the response analysis. Therefore an attempt was made to apply the two-level optimization technique to the same problem decomposing the original one into a number of smaller subproblems.

The sizing design variables $\boldsymbol{y}$ are then optimized for the fixed geometry at the lower level. Thereafter the geometry optimization with all the constraints are fulfilled at the top level coordinating the optimization process. Because of coupling, this procedure must be iteratively repeated until the optimum is reached (Figure 3).

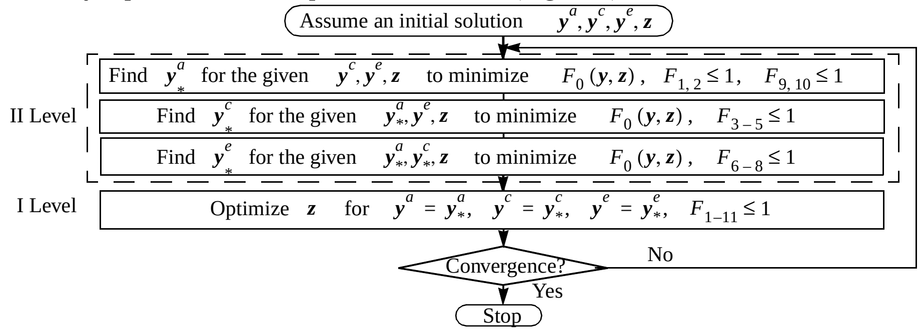

Figure 3. Flowchart for two-level optimization of the truck

We found it was essential that all the constraints should be satisfied (at least not strongly violated) before the geometry optimization, otherwise there might be no feasible solution of the coordination problem. That means that a proper choice of the constraints for the low-level optimization is very important.

5. Results

5.1. THE 2-D MODEL OF THE TRUCK

First the MPA optimizer along with the two-level procedure proposed were tested on a simplified problem with the 2-D model of the truck. All 20 design parameters (Table 1) were varied whereas the number of constraints were reduced to 8 (only limitations on the suspensions travels (6a)-(6c) were taken into account). To estimate the effectiveness of the two-level procedure the straightforward optimization was fulfilled too. The derivatives of the functions with respect to design parameters were not used during the optimization since we had found that taking them into account does not improve the convergence characteristics of the optimization process whereas the evaluation of one partial derivative and one response quantity are approximately of the same computational cost.

The optimization started from an infeasible design (maximum constraint violation of 60%). The two-level optimization was terminated after three cycles when there were no improvements in values of the objective function (Figure 5). Comparing the results (Table 1) it can be noted that the decomposed problem provided almost the same optimal design as the single-level optimization (1.270 and 1.262 m/s^2 respectively) but did not give considerable computational savings though (533 and 547 response analyses). Its usage might be justified for more complex 3-D problem with a large number of design variables and constraints when a straightforward optimization is difficult and even impossible.

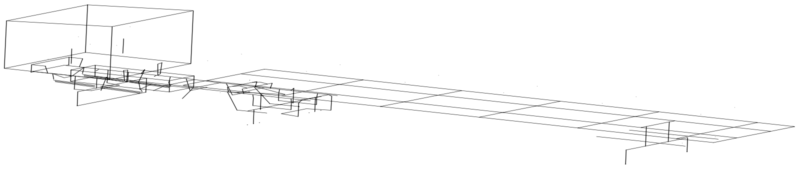

Figure 4. The 3-D finite element model of the truck and trailer combination

5.2. THE 3-D MODEL OF THE TRUCK AND TRAILER COMBINATION

To optimize the 3-D model of the truck and trailer combination (Figure 4) we used all the design variables and constraints (4a)-(4e).The response analysis for such a system was more sophisticated and time consuming. The FE model affected by 8 excitation inputs (corresponding to 8 wheels) contained about 2000 degrees of freedom. To obtain the ride index and the other response quantities the ANSYS program was externally coupled with TricaT post-processor developed at DAF Trucks NV.

TABLE 1. The results of the one- and two-level optimization of the truck

Design variable	2-D model					3-D model				Unit
	Definition			Result		Definition			Result	
	Lower bound	Upper bound	Initial value	Single level	Two levels	Lower bound	Upper bound	Initial value	Two levels	
y_1^a	400	2000	690	690	693	0.1	800	100	114	N/mm
y_2^a	1	50	35	24	23.5	0.5	25	16.9	7.7	Ns/mm
y_3^a	400	2000	600	1076	1079	100	500	150	100	N/mm
y_4^a	1	50	35	50	50	0.5	25	17	4.9	Ns/mm
y_1^c	50	160	90	100	104	25	80	45	45.5	N/mm
y_2^c	1	20	12.8	14.3	14.6	0.5	10	6.4	7.5	Ns/mm
y_3^c	50	185	150	163	157	25	92.5	75	74.4	N/mm
y_4^c	1	20	12.8	20	20	0.5	10	6.4	10	Ns/mm
y_5^c	8400	30000	8400	29567	27805	4200	20000	18000	20000	N/mm
y_1^e	7600	9900	7600	3590	3605	500	4950	2060	617	N/mm
y_2^e	8720	9900	8720	5650	5690	2500	4950	2600	2500	N/mm
y_3^e	24350	25000	24350	16754	16855	5000	12500	5400	6514	N/mm
z_1	1000	1250	1111	1000	1007	1000	1250	1111	1250	mm
z_2	100	600	600	583	584	100	600	600	537	mm
z_3	475	675	575	483	490	475	675	575	675	mm
z_4	700	1000	822	884	879	700	1000	822	700	mm
z_5	490	690	591	643	640	490	690	591	576	mm
z_6	890	915	906	893	895	890	915	903	910	mm
z_7	3250	3900	3250	3250	3250	3250	3900	3500	3900	mm
z_8	1100	1300	1201	1167	1170	1100	1300	1201	1206	mm
F_0	1.455			1.262	1.270			2.14	1.438	m/s^2
N iterations				26						
N analyses				547	533					

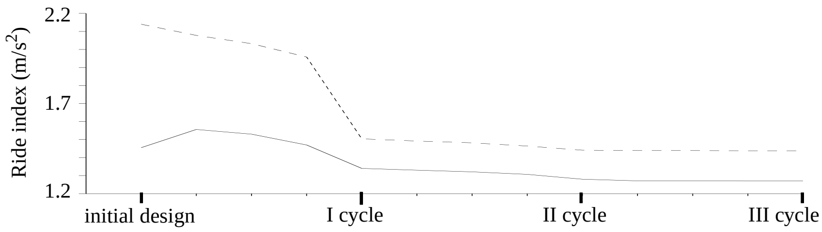

Figure 5. History of two-level optimization of the truck: —— 2-D model, – – – 3-D model

The optimization started then from a feasible design (the realistic model from DAF Trucks) for which the ride index was 2.140 m/s^2 (Table 1). The optimum design was obtained after three cycles with a reduction of the ride index to 1.438 m/s^2 (Figure 5). It should be noted that 2-D and 3-D model correspond to different trucks and the first one was used to test the optimization method only. From Figure 5 it is clear that changes in geometry strongly influence the objective function, and thus changes in the geometry represent a big reserve for further improvements of the existing vehicle.

6. Conclusions

The optimization of the dynamic characteristics of a linear mechanical system under stochastic load was done using a multipoint approximation technique. The results demonstrated that this technique could be efficiently used for dynamic problems. Moreover it can easily be coupled with a general-purpose finite-element software package.

Separating the sizing and geometrical design parameters the two-level optimization procedure for the above problem was proposed. Applied to the problem with the 2-D model of the truck it provided almost the same optimum design as the single-level optimization though it did not give considerable computational savings.

The results of optimization of the 3-D model of the truck and trailer combination showed robustness of the procedure for a complex systems with a large number of design variables and constraints.

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